Idiosyncratic Risk, Government Debt and Inflation

Matthias Hänsel*

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Abstract

How does public debt matter for price stability? If it is useful for the private sector to insure idiosyncratic risk, even transitory government debt expansions can exert upward pressure on interest rates and create inflation. As I demonstrate using a tractable model, this holds in the presence of an active Taylor rule and does not require the absence of future fiscal consolidation. Further analysis using a full-blown 2-asset HANK model reveals the quantitative magnitude of the mechanism to crucially depend on the structure of the asset market: under standard assumptions, the effect of public debt on the natural rate is either overly strong or overly weak. After disciplining this aspect based on pertinent evidence, my framework indicates relevant short-run effects of public debt on inflation under active monetary policy: In particular, in the HANK model the mechanism can account for US inflation remaining elevated in 2023 and afterwards.

Keywords: Monetary policy, Fiscal Policy, Inflation, HANK

JEL Classification: E31, E52, E63

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1 Introduction

In the aftermath of the Covid-19 pandemic as well as the economic shock following Russia's invasion of Ukraine, public debt levels have rapidly risen to historic highs in many advanced countries, particularly the USA. Should central banks be concerned about this? Standard macroeconomic models suggest that they should only if said debt is "unfunded", i.e., not backed by future government revenue. In the words of ECB board member Schnabel (2022), *"if governments do not credibly signal their commitment to responsible fiscal policies, the private sector may eventually expect that higher inflation is needed to ensure the sustainability of public debt"*. Otherwise, though, it doesn't need to affect the conduct of their policies.

However, the answer is more complex if government bonds have additional value for the private sector, e.g., as a means of insurance against idiosyncratic risk. In that case, public debt imperfectly crowds out aggregate demand and induces the private sector to require a higher real return if it is to hold more government debt: While already recognized as relevant for secular changes in interest rates (e.g., Rachel and Summers, 2019), this paper argues that public debt's ability to alter the "natural" and "neutral" rates of interest is also important for inflation and thus monetary policy in the short- and medium run.[1] In particular, elevated public debt can cause inflation even if a central bank pursues an active interest rate rule around the correct long-run natural rate and the country's fiscal authority is committed to raise enough surpluses to eventually pay back the debt (i.e., it is "funded"). All that is needed is the monetary authority not (or imperfectly) adjusting its reaction function in response to the government debt expansion.[2]

I first demonstrate this argument using a tractable New Keynesian model enriched with idiosyncratic income risk. As obtaining analytical results for such models is notoriously difficult in the presence of a positive net supply of assets, this framework naturally relies on many simplistic assumptions, such as households being ex-ante identical and subject to income risk only in a single period. However, this simplicity also has the virtue of clarifying that the mechanism does *not* rely on fiscal policy inducing any ex-ante redistribution towards constrained households, a channel that received substantial attention by the recent literature on Heterogeneous Agents New Keynesian (HANK) models. Additional assumptions elucidate that it does neither require the government to consume the resources it acquires through issuing debt nor is it relying on distortionary taxation to consolidate its finances, both of which could also affect inflation in models providing for

[1] The "natural" rate of interest is typically defined as the nominal rate consistent with a central bank's inflation target in the long term and taken to be "driven by fundamental factors in the economy, including demographics and productivity growth-the same factors that drive potential economic growth" (Powell, 2020). In contrast, the "neutral rate" is sometimes designated as the interest rate that would be consistent with target inflation in the short run (see e.g. Obstfeld, 2023).

[2] It is worth emphasizing that this *does not* mean the central bank not reacting at all. Indeed, I will always allow the monetary authority to react according to an interest rate rule satisfying the Taylor principle.

Ricardian equivalence. Furthermore, it is consistent with "active" monetary policy in the sense of Leeper (1991) and fiscal policy being committed to raise any amount of surplus necessary to pay back its debt.

Naturally, the analytical insights beget the question to what extent they may be quantitatively relevant: Encouragingly, a previous literature supports public debt exerting upward pressure on interest rates and I additionally find a positive association with inflation using US time series data. However, such reduced-form evidence is of limited help to isolate the effects of public debt in the short term, given that it doesn't vary by itself but is accompanied by fiscal policy measures that can themselves be inflationary.

Hence, I approach this issue by relying on a calibrated 2-asset HANK model. In this framework, households require liquid assets to insure themselves against skill- and business risk in the face of borrowing constraints but also have access to illiquid capital assets yielding higher returns. Besides ingredients relevant for a "neutral rate"-effect of public debt, it provides for other features that have been deemed important for the analysis of fiscal policy, such as the presence of Wealthy Hand-to-Mouth (HtM) households as well as empirically credible Marginal Propensities to Consume (MPCs).

However, under different assumptions on the structure of its asset market, it can generate disparate relationships between government debt supply and interest rates: If liquid and illiquid assets are traded on *segmented* markets, as e.g. in Kaplan et al. (2018) or Bayer et al. (2024), then higher government debt supply increases liquid bond rates *much more* in the long run than suggested by the aforementioned evidence. In contrast, if both bonds and capital can be freely held either as liquid or illiquid asset, as e.g. in Auclert et al. (forthcoming), then more public debt is associated with a *much weaker* rise in rates. As I explain, this tension is tightly connected to a key calibration margin for 2-asset HANK models and thus unlikely to be a peculiarity of my setup. As a resolution, I propose a simple extension that enables to move in between these two polar cases and discipline it based on previous evidence.

With the suitably calibrated model, I then analyze how government bond supply affects the time path of inflation in response to a simple fiscal policy shock. Leveraging my setup for the model's asset market, I find public debt's potential to impact medium- to long run interest rates to be a key mediator of the ensuing price level changes. In my baseline, the respective effects are immediate, moderate in absolute magnitude but potentially quite persistent. A decomposition of the household sector's partial equilibrium response furthermore identifies investment demand as the crucial aggregate demand margin behind the price pressure. This suggests that while simple models abstracting from capital may indicate potential effects of public debt on the "natural" rate and inflation, they are likely too simple for a serious quantitative assessment thereof.

After further robustness checks, I ask what role the inflationary pressure exerted by public debt might have played for the recent US inflation experience: For this purpose, I extend the HANK model with an Effective Lower Bound (ELB) and filter a series of business cycle shocks that make it align with the aggregate US data post-2020. The exercise reveals that while the "debt inflation" is unlikely to have played a big role for generating the inflation peak in 2022, it can quantitatively explain inflation remaining elevated in 2023 and afterwards. As an auxiliary insight, it supports the view of Giannone and Primiceri (2024) that the US post-Covid inflation was mostly driven by "demand-side" factors.

Finally, I briefly consider potential implications for monetary policy: By explicitly adjusting nominal rates in response to the value of public debt, central banks can avoid the "debt inflation" and achieve comparatively faster disinflation at lower nominal interest rates: However, this also requires that component of its reaction function to be understood by the private sector. A "Difference rule" as proposed by Orphanides and Williams (2002) may involve substantial economic costs if it is parameterized similar to a Taylor rule or subject to monetary policy shocks.

1.1 Related Literature

On the one hand, this paper connects to a long tradition in macroeconomics studying monetary-fiscal policy interactions, going back to the seminal works of Sargent and Wallace (1981) and Leeper (1991). Leeper and Leith (2016) and Cochrane (2023) offer summaries of this literature, including its modern incarnation as Fiscal Theory of the Price Level (FTPL). An import recent contributions is the work by Bianchi et al. (2023), who find fiscal policy important to explain inflation persistence in the US and also study the US post-pandemic inflation. Additionally, Kaplan et al. (2023) study the FTPL in a heterogeneous agent setting featuring uninsurable idiosyncratic risk. As already indicated above, most these works differ from mine in that they focus on the inflationary effects of *unfunded* government debt (i.e. not backed by future surpluses).

On the other hand, my paper is part of the sprawling HANK literature: Here, it particularly relates to papers studying fiscal policy using two-asset models such as Auclert et al. (forthcoming), who emphasize the importance of MPCs, and Bayer et al. (2023a), who focus on the effects of public debt supply on interest rates and investment. While these papers mostly analyze the real effects of fiscal policy, my model builds on the frameworks used in these works. Additionally, my mentioned results on the asset market structure indicates that under their respective assumptions, these aspects can likely not be studied jointly. Other HANK research on fiscal policy such as Hagedorn et al. (2019) or Seidl and Seyrich (2023) also mostly restrict attention to its real effects. An exception is the recent work of Angeletos et al. (2024), who study to what extent HANK models and representative agent models featuring the FTPL can generate the same inflation responses. However, they effectively abstract from the mechanism highlighted in this paper by assuming that monetary policy sets real interest rates directly.

Moreover, my results on the importance of the asset market structure for liquid rate dynamics bear some resemblance to the findings by Chiang and Żoch (2023), who study a

2-asset HANK model with explicit financial intermediation. Comparing their structure with alternative settings, they find the calibration of the asset market to be important for the real effects of different policy shocks. However, they do not consider inflation as an outcome and also assume real returns on liquid assets to be fixed.[3]

Naturally, my work also connects to a set of previous studies analyzing fiscal policy in other settings deviating from Ricardian Equivalence. In particular, related inflationary effects of "funded" government debt were also noticed by Ascari and Rankin (2013) and Aguiar et al. (2023) in the context of Overlapping Generations (OLG)-models with nominal rigidities and by Linnemann and Schabert (2010) in a New Keynesian framework assuming that public debt provides transaction services. Besides building on a different micro-foundation, my work also employs a richer, more quantitatively oriented model.

Finally, after a previous version of this paper was first circulated, Campos et al. (2024) released an independently developed working paper that analyzes the effects of permanent public debt expansions on the natural rate in a simple HANK-type economy without capital. Amongst others, they argue that a monetary rule proposed by Orphanides and Williams (2002) might a be suitable tool for ensuring price stability in their setting. My analysis indicates their model to miss out on a crucial investment demand margin intermediating the effects of public debt supply on interest rates and inflation and suggests that the Orphanides and Williams (2002)-rule may have downsides in richer settings.

The remainder of the paper is structured as follows: Section 2 presents the tractable New Keynesian model enriched with income risk and derives the results referred to above. Section 3 provides further intuition and discusses some empirical considerations regarding the relationship between public debt, (liquid asset) interest rates and inflation. Section 4 then presents the 2-asset HANK model used for the quantitative analysis, the calibration of which is detailed in Section 5. Section 6 elaborates on how different asset market structures shape to what extent public debt can affect the long-run natural rate in HANK models. Insights from this section will in turn be used in Section 7 to pinpoint the inflationary effects of public debt itself, the results from which will be subjected to various robustness checks in Section 8. Section 9 contains the results of the "debt inflation" channel for the US inflation before Section 10 briefly discusses some implications for monetary policy. Section 11 concludes.

2 An Analytical Model

This section presents a simple New Keynesian model enriched with idiosyncratic income risk, in which it is possible to analytically characterize the mechanism mentioned above.

[3] Presumably, their insights are thus the reverse of my mine: If changing liquidity supply would necessitate substantial interest rate adjustments for asset market clearing but this is prevented, then other parts of the economy have to adjust strongly.

2.1 Model setup

Time is discrete and runs forever, starting from $t = 0$. There is no aggregate uncertainty, but households face idiosyncratic income risk as specified below.

2.1.1 Households

The model is inhabited by a unit mass of *ex-ante identical* households (also referred to as "agents" below), which gain utility from consumption and leisure according to the utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log(c_{it}) + \gamma \log(1 - N_{it})\right] \quad ,$$

where N_{it} denotes time worked. The period felicity function corresponds to the same analytically convenient balanced growth preferences as used by Aguiar et al. (2023). Furthermore, it is assumed that in $t = 0$, each individual has the same labor productivity $z_0 = 1$, i.e. they supply one efficiency unit of labor per unit of time worked.
Between periods 0 and 1, and at that time only, households face *idiosyncratic* income risk. In particular, they transition to a state of high labor productivity $z^h > 1$ with probability ρ^h and to a state of low labor productivity $z^l < 1$ with probability $\rho^l = 1 - \rho^h$. These labor productivities remain fixed for $t \geq 1$ onwards, so as of that time, there will be a fraction ρ^h of "high productivity" households and a fraction ρ^l of "low productivity" households. For tractability, I restrict

$$\rho^h z^h + (1 - \rho^h) z^l = 1 \quad , \tag{1}$$

so that the economy's average labor productivity is not affected by the time 0 risk.
In any period, a household with productivity $i \in \{h, l\}$ faces the budget constraint

$$P_t w_t z^i N_t^i + (1 + i_t) B_{t-1} + P_t T_t = P_t c_t + P_t z_t \tau_t + B_t \quad ,$$

which can be stated in real terms as

$$w_t z_t N_t^i + \frac{1 + i_t}{\pi_t} b_{t-1} + T_t = c_t + z_t \tau_t + b_t \tag{2}$$

where $b_t := B_{it}/P_t$ and $\pi_t := P_t/P_{t-1}$. P_t denotes the current price level, w_t the real wage and T lump-sum transfers from the government. B_t denotes holdings of nominal bonds that each yield a gross nominal return of $1 + i_t$. Additionally, the government may levy a non-distortionary tax proportional to individual labor productivity, which is denoted by τ. I additionally impose that in period 0, each household starts out without any bonds, i.e., $B_{-1} = 0$: This is not only analytically convenient, but also clarifies that, unlike for the FTPL, none of the results derived here rely on "surprise" asset revaluations (cf. Niepelt, 2004).

2.1.2 Final good firms

The economy's final good is produced by a representative firm, which combines intermediate goods $y_t(j)$ according to the following CES production function:

$$Y_t = \left[\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}} \tag{3}$$

Taking prices of intermediate goods $p_t(j)$ as given, the firm's optimization problem implies it will demand $y_t(j)$ according the familiar demand structure

$$y_t(j) = \left[\frac{p_t(j)}{P_t}\right]^{-\epsilon} Y_t \quad , \tag{4}$$

resulting in the price of its final good to be

$$P_t = \left[\int_0^1 p_t(j)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}} \quad .$$

2.2 Intermediate good firms

Intermediate goods are produced by a unit mass of firms, each of which produce a single variety j as monopolists using labor purchased from households at real wage w_t, which they take as given.

For simplicity, it is assumed that the intermediate goods firms are owned by risk-neutral "capitalists" who cannot participate in the bond market and discount the future at the same rate β as the households.[4] Similar assumptions are common for so-called "tractable HANK" models in the literature and aim to reduce the dependence of household behavior on firm profits (e.g., Broer et al., 2020).

The intermediate goods firms are endowed with an identical initial price level $p_{-1}(j) = P_{-1}$ and face a quadratic price adjustment cost à la Rotemberg (1982), subject to which they maximize

$$\sum_{t=0}^{\infty} \mathbb{E}_0 \beta^t \left[(p_t(j) - w_t) \left[\frac{p_t(j)}{P_t}\right]^{-\epsilon} Y_t - \frac{\phi}{2}\left(\frac{p_t(j)}{p_{t-1(j)}} - 1\right)^2 Y_t \right] \quad .$$

The first order conditions of the price-setting problem in period t is

$$(1-\epsilon)\left[\frac{p_t(j)}{P_t}\right]^{-\epsilon} Y_t + \epsilon \frac{w_t}{p_t(j)}\left[\frac{p_t(j)}{P_t}\right]^{-\epsilon-1} Y_t - \phi\left(\frac{p_t(j)}{p_{t-1(j)}} - 1\right)\frac{1}{p_{t-1}(j)}$$
$$+ \beta \mathbb{E}_t \left(\frac{p_{t+1}(j)}{p_{t(j)}} - 1\right)\frac{p_{t+1}(j)}{(p_t(j))^2} = 0$$

and by restricting focus to a symmetric equilibrium so that $p_t(j) = P_t$, we obtain the following New Keynesian Phillips curve:

$$\phi(\pi_t - 1)\pi_t = (1-\epsilon) + \epsilon w_t + \beta \mathbb{E}_t \frac{Y_{t+1}}{Y_t}\phi(\pi_{t+1} - 1)\pi_{t+1} \quad . \tag{5}$$

[4]Specifying a different discount factor for the firms would not affect any of the results below.

2.2.1 Government

The government consists of two branches, a *monetary authority* and a *fiscal authority*. The *monetary authority* determines the nominal interest rate according to the standard Taylor rule

$$i_{t+1} = r_t^* + \theta_\pi(\pi_t - 1) \quad , \tag{6}$$

where r_t^* is taken as a parameter. This is consistent with typical "textbook" formulations as e.g. in Galí (2015) but allows to account for the neutral period being different in $t = 0$. I restrict $r_t^* = \frac{1}{\beta} - 1 \ \ \forall t > 0$, which will below be shown to be the neutral rate of interest once idiosyncratic risk has been resolved.

The *fiscal authority* can provide lump-sum transfers to households, which are financed by issuing nominal government bonds B^g or levying taxes τ. In real terms, the budget constraint of the fiscal authority is thus

$$T_t + \frac{1+i_t}{\pi_t} b_{t-1}^g = b_t^g + \tau_t \int_0^1 z_t(i) di \quad . \tag{7}$$

For the analysis, I focus on the following time path of fiscal policy: The fiscal authority starts without initial debt, $b_{-1}^g = 0$. In $t = 0$, the government pays out a lump-sum transfer to households that is entirely financed by debt, i.e. $T_0 = b_0^g$ and $\tau_0 = 0$. In $t = 1$, the government pays back all the debt, which requires taxes $\tau_1 = \frac{1+i_1}{\pi_1} b_0^g$. Afterwards, the fiscal authority remains inactive, $b_t^g = 0$, $T_t = 0$ as well as $\tau_{t+1} = 0 \ \forall t \geq 1$.[5]

Note that the initial transfer does not involve *any* ex-ante redistribution, as households are homogeneous in period 0. Additionally, it is obvious that in this setting all government debt is backed by future surpluses, since the fiscal authority will raise any amount of taxes necessary to pay back the debt in period 1.

2.3 Equilibrium Analysis

I begin with characterizing the equilibrium for the periods $t \geq 1$, during which there is no more idiosyncratic risk and the government chooses $b_t^g = 0$:

Proposition 1. *For $t \geq 1$, the equilibrium is characterized by the following*

- *aggregates:*

$$\begin{aligned} \pi_t = \pi_{ss} = 1, \quad w_t = w_{ss} = \frac{\epsilon - 1}{\epsilon}, \\ i_{t+1} = i_{ss} = \frac{1-\beta}{\beta}, \quad Y_t = N_t = N_{ss} = \frac{1}{1+\gamma} \end{aligned} \tag{8}$$

[5]In the model, Ricardian equivalence will effectively hold once the idiosyncratic risk has been resolved by $t = 1$. Thus, the assumption of inactive fiscal policy from period 1 onward can be relaxed as long as the uniform lump-sum transfers are not high enough to completely insure the initial income risk.

- *Household policies:*

$$c_t^i = \frac{1}{1+\gamma}\left(w_{ss}z^i + \frac{i_{ss}}{1+i_{ss}}\left((1+i_1)b_0 - z^i\tau_1\right)\right) := c_{ss}^i \quad \forall i \in \{l,h\} \tag{9}$$

$$N_t^i = \frac{1}{1+\gamma}\frac{1}{w_{ss}z^i}\left(w_{ss}z^i - \gamma\frac{i_{ss}}{1+i_{ss}}\left((1+i_1)b_0 - z^i\tau_1\right)\right) := N_{ss}^i \quad \forall i \in \{l,h\} \tag{10}$$

Proof. See Appendix A.1. □

At time 1, all uncertainty is resolved and the only aggregate state fixed after b_0^g was paid back.[6] Hence, the economy enters a steady state at this point.

Using the above results, we can now characterize the equilibrium in period 0:

Proposition 2. *In period* 0*, we have*

$$Y_0 = N_0 = \frac{1}{1+\gamma}\,,\; c_0 = \frac{w_0}{1+\gamma} \quad \textit{as well as} \quad w_0 = \frac{\phi(\pi_0-1)\pi_0 + \epsilon - 1}{\epsilon} \tag{11}$$

while the rate of inflation is implicitly characterized by

$$\begin{aligned}\frac{\epsilon}{\epsilon - 1 + \phi(\pi_0-1)\pi_0} = \beta\rho^h & \frac{1 + r_0^* + \theta_\pi(\pi_0-1)}{w_{ss}z^h + \frac{i_{ss}}{1+i_{ss}}\left(1 + r_0^* + \theta_\pi(\pi_0-1)\right)b_0^g(1-z_h)} \\ &+ \beta(1-\rho^h)\frac{1 + r_0^* + \theta_\pi(\pi_0-1)}{w_{ss}z^l + \frac{i_{ss}}{1+i_{ss}}\left(1 + r_0^* + \theta_\pi(\pi_0-1)\right)b_0^g(1-z^l)} \quad . \end{aligned} \tag{12}$$

Proof. See Appendix A.2. □

Labor supply is constant in period 0, regardless of the realized wage rate: This is a consequence of the chosen preferences, which imply that income- and substitution effects of a wage change offset each other. So, as in Aguiar et al. (2023), inflation and wage changes do not affect the level of output, but only redistribute between households and the owners of intermediate goods firms.

From (12), we immediately obtain the following result regarding the "natural" rate of interest r_0^n under which $\pi_0 = 1$:

Lemma 1. *In period* 0*, the natural rate of interest* r_0^n *is implicitly characterized by*

$$\begin{aligned}\frac{\epsilon}{\epsilon-1} = \beta\rho^h & \frac{1 + r_0^n}{w_{ss}z^h + \frac{i_{ss}}{1+i_{ss}}\left(1 + r_0^n\right)b_0^g(1-z^h)} \\ &+ \beta(1-\rho^h)\frac{1 + r_0^n}{w_{ss}z^l + \frac{i_{ss}}{1+i_{ss}}\left(1 + r_0^n\right)b_0^g(1-z^l)} \quad . \end{aligned} \tag{13}$$

Equation (13) indicates that the natural rate of interest will in general depend on the level of government debt b_0^g. While the functional forms do unfortunately not provide for a closed-form solution, the implicit function theorem allows us nevertheless to arrive at the following result:

[6] Note that the model does not feature price dispersion due to the assumption of Rotemberg (1982) pricing.

Proposition 3. *Assume* $b_g^0 \in \left[0, \frac{\epsilon-1}{\epsilon}\frac{\beta}{1-\beta}\right)$. *In that case,*

$$\frac{\partial r_0^n}{\partial b_0^g} > 0 \quad ,$$

i.e. the natural rate of interest is increasing in the level of government debt issued.

Proof. See Appendix A.3. □

The above result tells us that in the initial period featuring idiosyncratic income risk, the natural rate of interest indeed increases in the level of government debt, at least under some mild restrictions on the amount of the latter.[7] The reverse of this result is that if the government issues more debt without the central bank adjusting the intercept of its Taylor rule, inflation ensues. Formally:

Proposition 4. *Assume that* r_0^* *is fixed at the natural rate* $r_0^n(\bar{b})$*, as implicitly defined by* (13)*, for some given level* $\bar{b} \in \left[0, \frac{\epsilon-1}{\epsilon}\frac{\beta}{1-\beta}\right)$ *of government debt to be issued in period 0. Then,*

$$\left.\frac{\partial \pi_t}{\partial b_0^g}\right|_{b_0^g=\bar{b}} > 0 \quad ,$$

i.e. inflation increases in the amount of government debt.

Proof. See Appendix A.4. □

Summing up, from the simple model above we learned the following: If households face idiosyncratic income risk, increases in government debt raise the "natural" or "neutral" rate of interest. Correspondingly, the central bank would need to adjust its interest rule to the amount of government debt if it wants to avoid inflation. As is clear from the model structure, these results required neither the government consuming the resources nor them being used for any *ex-ante* redistribution. Of course, it requires some *ex-post* redistribution, so that the debt issued can actually serve insurance purposes: If the fiscal authority would repay its debt by raising uniform lump-sum taxes instead, any type $i \in \{h, l\}$ household would be taxed exactly equal to their savings in $t = 1$ and the total amount of debt be irrelevant for inflation.

3 Simple Intuition and Empirical Considerations

At this point, it may be helpful to provide further intuition for *why* elevated government debt influences inflation in the presence of a Taylor rule. Assume that indeed, as in the

[7] If the amount of government debt issued becomes too high, the proportional tax rule would eventually eliminate the difference between h and l worker consumption from period 1 onwards. However, under typical calibrations of New Keynesian models, that level would be very high. Typically, ϵ would be at least 6 and β greater than 0.95.

model above, the "natural" gross interest rate $\tilde{R}$ prevailing in an economy depends on the amount of government debt B^g in circulation. Assume furthermore that the economy's central bank aims to stabilize inflation around a target π^* by setting the nominal interest rate i_t according to a Taylor rule of the form

$$1 + i_t = \pi^* R^* + \theta_\pi(\pi_t - \pi^*) \quad . \tag{14}$$

This is a version of (6) allowing for a positive net inflation target. R^* denotes the natural (gross) rate consistent with some initial level of government bonds B_0^g. Now, the amount of government debt in circulation rises temporarily to $B_1^g > B_0^g$. Notice that we can add and substract a term $\tilde{R}(B_1^g)$ to (14) and re-write it as

$$1 + i_t = \tilde{\pi}_1 \tilde{R}(B_1^g) + \theta_\pi(\pi_t - \tilde{\pi}_1) \ \text{ with } \ \tilde{\pi}_1 := \pi^* \frac{\theta_\pi - R^*}{\theta_\pi - \tilde{R}(B_1^g)} \quad . \tag{15}$$

If $\tilde{R}(B_1^g) > R^*$, then $\tilde{\pi}_1 > \pi^*$. So, if public debt rises and the central bank sticks to rule (14) and the "true" natural rate $\tilde{R}$ that may depend on debt is de facto higher than R^*, the Central Bank would seem to operate as if having a higher "implicit" inflation target. It is also helpful to look at the effect through the Fisher equation, which, under perfect foresight, states the real liquid asset return r_t^{real} to fulfill

$$1 + r_{t+1}^{real} = \frac{1 + i_t}{\pi_{t+1}} = \frac{\pi^* R^* + \theta_\pi(\pi_t - \pi^*)}{\pi_{t+1}} \quad . \tag{16}$$

If r_{t+1}^{real} needs to be higher for the public to be willing to hold a certain amount of public debt, the right-hand side (RHS) terms need to be as well. Substituting i_t using the Taylor rule, we see that this requires either π_t to be higher or π_{t+1} to be lower. Now, if both are linked through a forward-looking New Keynesian Phillips curve such as (5) and $\theta_\pi > 1$, the adjustment will typically not be able to work through a lower π_{t+1}, as this would also lower π_t and decrease the nominator of (16) even more. Hence, for a active interest rule only reacting to inflation, higher public debt supply must cause higher inflation.[8]

3.1 Empirical Considerations

As demonstrated by the previous analysis, government debt can be inflationary under active monetary policy if it induces demand pressure and raises the neutral interest rate. While it will become clear below that the actual realization of inflation also depends on central bank policy, a natural pre-condition for there to be any effects is that increasing public debt actually exerts upward pressure on liquid asset returns. Reassuringly, this aligns well with an empirical literature attempting to measure the effects of public debt on treasury rates empirically, exemplified e.g. by Engen and Hubbard (2005) or Laubach (2009). According to a summary in Rachel and Summers (2019), such estimates indicate

[8] Having the rule reacting to output wouldn't generally resolve the issue: In the simple model above, public debt was inflationary despite the output gap being always 0.

medium- to long-term effects of 3 and 6 basis points (bp) per percentage point increase in the Debt-to-GDP ratio.

As suggestive evidence that public debt supply is also associated with higher real interest rates and inflation in the shorter run, I additionally run a series of Local Projections (Jordà, 2005) of the form

$$y_{t+h} = \beta_h^0 + \beta_h^t t + \beta_h B_t + \Gamma_h Z_{t-1} + u_{t+h} \quad ,$$

where B_t denotes the beginning-of-period/end-of-previous-period market value of public debt relative to GDP, t a time trend and Z_t a series of control variables that always contain four lags of the unemployment rate, inflation, federal government expenditures (relative to GDP), the 5-year expected real return on treasuries, B_t as well as the dependent variable y. Details on the construction of the variables are provided in Appendix C. The sample is quarterly, starts in 1982 and I end it at the end of 2019 to make clear that the results are not driven by the outliers during 2020-2022.[9]

The resulting parameter estimates for β_h are displayed in Figure 1: In Panel (a), we see that an innovation to public debt may potentially be quite persistent, but the estimates get quite noisy after a few quarters. Panel (b) plots the corresponding response of the expected real returns on 5-year treasury bonds, which again features wide confidence intervals but is consistent with rising liquid asset returns of a magnitude comparable to the long-run results. In the bottom panels, we additionally notice that both medium run inflation expectations and actual inflation increase after the innovation to public debt. All in all, the empirical responses seem consistent with public debt exerting moderate upward pressure on interest rates and inflation, although it should be emphasized that they cannot prove it: The value of government debt doesn't move by itself but in response to public spending or events inducing bond revaluation, either of which might be the true underlying cause of the dynamic correlations.

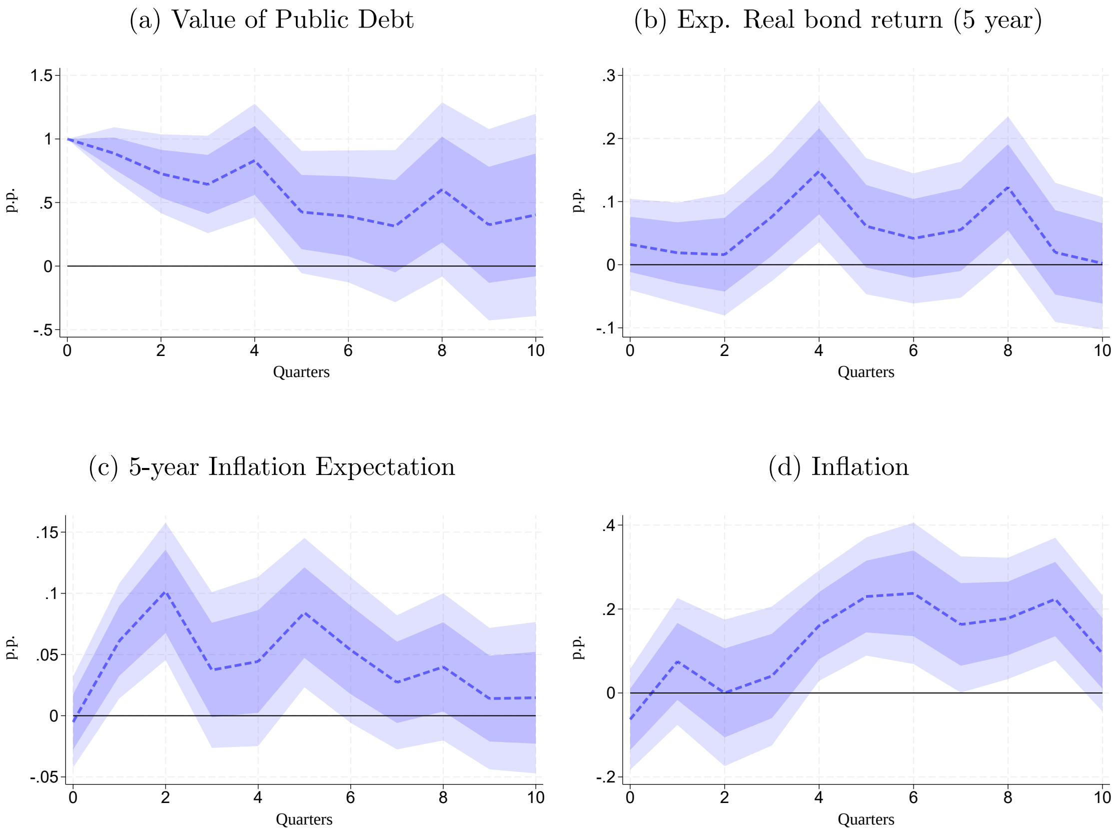
It is additionally worth mentioning that recent empirical studies indicate more clearly identified fiscal expansions to also be linked to subsequent inflation (e.g., Jordà and Nechio, 2023; Hazell and Hobler, 2024). However, even such reduced-form evidence does not allow discriminating between different channels: For example, to the extent that fiscal shocks redistribute to constrained households', they can affect inflation even if the natural rate were independent of public debt, as e.g. in so-called Two-Agent New Keynesian (TANK) models.

3.2 Is the channel plausibly relevant?

Given the above discussion and results, one should now ask whether the public debt-induced pressure on the "natural" rate might plausibly be *quantitatively* relevant for in-

[9]Extending the sampe up to 2024 would just result in me finding stronger correlations. The sample start period is due to me using the Cleveland Fed's inflation expectation estimates (based on Haubrich et al., 2012) to compute expected real returns, which is not available before 1982.

Figure 1: Results of Local Projection estimates



Note: Shaded areas indicate 68- and 90% confidence bands using Robust Standard Errors (Montiel Olea and Plagborg-Møller, 2021). All responses are scaled as percentage points (p.p.), in case of public debt as a fraction of GDP.

flation dynamics. After all, a 3-6 basis points increase in real rates per percentage point increase in the debt-to-GDP ratio doesn't seem particularly much and public debt's association with inflation could also reflect effects of the expenditure it funds. To get a grasp on this, (15) suggests a simple "back-of-the-envelope" calculation: Choosing some plausible values for R^*, θ_π and π^*, we can compute how much potentially debt-driven deviations of $\tilde{R}(B^g)$ from R^* affects $\tilde{\pi}$, the "implicit" inflation target derived above.

Let us assume the interest rule to be operated on a quarterly basis with an intercept of $\theta_\pi = 1.5$, a standard value going back to Taylor (1993), and that $R^* = 1.02^{1/4}$ and $\pi^* = 1.02^{1/4}$, i.e. both the intercept in the central bank's rule corresponds as well as its inflation target are 2% annually. Now, if the annual debt-to-GDP ratio has gone up by 10% and the annual $\tilde{R}(B^g)$ linearly increases by 4 basis points for each of these percentage points, we will have $\tilde{R}(B^g) = (1.02 + 10 \times 0.0004)^{1/4}$. For these values, we then find the annualized "implicit" inflation target to be

$$\tilde{\pi}^4 = \left(\pi^* \frac{\theta_\pi - R^*}{\theta_\pi - \tilde{R}(B_1^g)}\right)^4 \approx 1.0281 \quad .$$

In words, in that case the "implicit" target amounts to 2.8% instead of 2%. We can thus conclude that while it seems unlikely that just the "natural" rate pressure stemming from an elevated amount of debt itself is unlikely to have caused inflation peaks of the magnitudes observed in 2022, such a magnitude is consistent with the persistent "last mile" inflation observed afterwards.

4 The Quantitative HANK model

While analytically tractable models as e.g. in the previous Section 2 allow for a sharp characterization of the effects of government policy, their simplicity only admit a qualitative assessment of the mechanism at hand. Additionally, reduced-form evidence provides little information on the underlying channels and the simple "back-of-the-envelope" calculation as in the previous Section 3.2 abstracts from potentially important general equilibrium considerations such as the effects of nominal rigidities. In turn, it is necessary to employ a quantitative model to figure out to what extent public debt may affect inflation through its usefulness for the private sector. To this end, I employ a 2-asset HANK model, most features of which are deliberately similar to frameworks in the previous literature, e.g., Bayer et al. (2024) and Auclert et al. (forthcoming). While a 2-asset set-up is not strictly necessary for public debt to create inflation via the liquidity channel forwarded by this work, a serious quantitative investigation should arguably feature a somewhat realistic level and distribution of aggregate wealth and be consistent with consumption behavior that was found important for studying fiscal policy in previous work. Typical 1-asset model can't provide for these features (Kaplan and Violante, 2022). Additionally, it will turn out that limited suitability of capital for providing liquid assets and the resulting time-varying liquidity premia will be key for the quantitative magnitude of the results. To allow for flexibility in this regard, my model deviates from previous work regarding how a financial intermediary referred to as the *liquid asset fund* below is modelled.

4.1 Households

4.1.1 Idiosyncratic states

There is a unit mass of households, which I again also refer to as "agents" interchangeably. These differ ex-post by several idiosyncratic states:

- First of all, households vary in terms of their holdings of liquid and illiquid assets a_{it} and k_{it}. k_{it} represents holdings of capital and I require that $k_{it} \geq 0$ as well as $a_{it} \geq \underline{a}$, with $\underline{a}$ representing an exogenous borrowing/short-selling limit. Capital is illiquid in that a household can change her stock k_{it} only infrequently: In particular, following Bayer et al. (2024) and Auclert et al. (forthcoming), I assume that the opportunity to do so arises randomly in an i.i.d. fashion, in that households only gets to participate in the market for illiquid assets with probability $\lambda \in (0, 1)$ every period.

- Secondly, the agents can be workers ($\Xi_{it} = 0$) or "entrepreneurs" ($\Xi_{it} = 1$). The former participate in the regular labor market, while the latter don't supply labor to the market but receive the profits generated by the firms (to be described below), which, for simplicity, are assumed to be shared equally among all households with $\Xi_{it} = 1$. Transitions to and out of the "entrepreneur" status are exogenous with probabilities ζ and ι, implying a time-invariant mass of $m^{\Xi} := \frac{\zeta}{\zeta+\iota}$ agents in that state.

- Worker households ($\Xi_{it} = 0$) additionally differ by their idiosyncratic labor productivity or "skill" $s_{it} \in \mathcal{S} = \{s_1, s_2, ..., s_{ns}\}$, which evolves stochastically according to a discrete Markov chain with transition probabilities $\pi^s(s_{it+1}|s_{it})$. Workers who are selected to become entrepreneurs lose their idiosyncratic s_{it} as exiting entrepreneurs draw a new s_{it} from the ergodic distribution of the Markov Chain.

4.1.2 The Household problem

Households gain utility from consumption c and disutility from its amount of hours worked according to the preference structure

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \prod_{\tau=0}^{t} (\beta A_\tau) \left(\frac{c_{it}^{1-\xi} - 1}{1-\xi} - \varsigma \frac{N_t^{1+\gamma}}{1+\gamma} \right) \quad . \tag{17}$$

The above formulation allows for a time-varying demand shock A_t shifting all households' discount factor β, which will be used to induce consumption restraints in Section 9.[10] As e.g. in Auclert et al. (forthcoming), households are *not* free to choose their own labor supply. Instead, they are required to work the number of hours demanded by their employers at the wage determined by a set of labor unions as detailed below. These will be equal in equilibrium, i.e., $N_{it} = N_t \; \forall i \in (0, 1)$.

An agent who gets to adjust her illiquid capital stock will the face budget constraint

$$c_{it} + q_t k_{it+1} + a_{it+1} = y_{it}(s_{it}, \Xi_{it}) + (1 + r^a_t(a_{it}))a_{it} + (q_t + r^k_t)k_{it} + T_{it} \tag{18}$$

while for non-adjusters, the constraint will be of the form

$$c_{it} + a_{it+1} = y_{it}(s_{it}, \Xi_{it}) + (1 + r^a_t(a_{it}))a_{it} + r^k_t k_{it} + T_{it} \quad . \tag{19}$$

Both budget constraint are already written in real terms. q_t denotes the time t price of capital goods, T_{it} a transfer from the government, r^k_t the real net return of capital goods and $r^a_t(a_{it})$ the real return on bonds a_{it}. The latter depends on a_{it} due to the presence of a borrowing penalty. In particular, we have

$$r^a_t(a_{it}) = \begin{cases} r^l_t & \text{if } \; a_{it} \geq 0 \\ r^l_t + \bar{R} & \text{if } \; a_{it} < 0 \end{cases} \tag{20}$$

where r^l_t is the real return on liquid savings, which will depend on the nominal central bank rate r^R_t and inflation $\pi_t = \frac{P_t}{P_{t-1}}$ as specified below. $\bar{R}$ is a real borrowing penalty.[11] Finally, y_{it} represents a household's post-tax labor- or profit income which is given by

$$y_{it}(e_{it}, s_{it}, \Xi_{it}) = \begin{cases} (1 - \tau^w_t)\,(w_t s_{it} N_t)^{1-\tau^p} & \text{if } \;\; \Xi_{it} = 0 \\ (1 - \tau_t \Xi)\frac{\Pi_t}{m^{\Xi}_t} & \text{if } \;\; \Xi_{it} = 1 \end{cases} \quad . \tag{21}$$

Labor income is subject to an affine tax schedule in the veins of Benabou (2002) for which the parameters τ^w and τ^p determine the level and degree of progressivity, respectively. Similarly, τ^{Ξ} is the tax rate on entrepreneurs' profit income. Both level parameters may be adjusted by the government over time and thus have time subscript.

Letting Γ_t denote a set containing the economy's *aggregate* state at period t, we are now ready to state the Bellman equation corresponding to the households' dynamic utility maximization problem, which are

$$V^a(a_{it}, k_{it}, s_{it}, \Xi_{it}; \Gamma_t) = \max_{c_{it}, k_{it+1}, a_{it+1}} \left\{ \frac{c_{it}^{1-\xi} - 1}{1 - \xi} - \varsigma \frac{N_t^{1+\gamma}}{1+\gamma} + \beta \mathbb{E}_t A_{t+1} V(a_{it+1}, k_{it+1}, s_{it+1}, \Xi_{it+1}; \Gamma_{t+1}) \right\}$$
$$\text{s.t. to (18), (21), } k_{it} \geq 0 \text{ and } a_{it} \geq \underline{a} \tag{22}$$

[10]This is in line with Bardóczy et al. (2024), who also use discount factor shocks in a HANK study relating to the US post-2020 period.

[11]My specification for the borrowing wedge implies that every unit of debt held by a household incurs a real resource cost of $\bar{R}$, e.g. due to costly monitoring.

for an household able to adjust its capital stock and

$$V^{na}(a_{it}, k_{it}, s_{it}, \Xi_{it}; \Gamma_t) = \max_{c_{it}, a_{it+1}} \left\{ \frac{c_{it}^{1-\xi} - 1}{1-\xi} - \varsigma \frac{N_t^{1+\gamma}}{1+\gamma} + \beta \mathbb{E}_t A_{t+1} V(a_{it+1}, k_{it}, e_{it+1}, \Xi_{it+1}; \Gamma_{t+1}) \right\}$$

$$\text{s.t. to (19), (21), } k_{it} \geq 0 \text{ and } a_{it} \geq \underline{a} \tag{23}$$

for an household that unable to do so. The ex-ante value function $V(\cdot)$ is given by

$$V(a_{it+1}, k_{it}, e_{it+1}, s_{it+1}, \Xi_{it+1}; \Gamma_{t+1}) = \lambda V^a(a_{it}, k_{it}, e_{it}, s_{it}, \Xi_{it}; \Gamma_t) + (1-\lambda) V^{na}(a_{it}, k_{it}, e_{it}, s_{it}, \Xi_{it}; \Gamma_t) \quad .$$

4.2 Production

The model's supply side is similar to standard "medium scale" DSGE models: Production is vertically integrated. There is again a final good that can either be consumed or used by capital goods producers to produce investment goods subject to adjustment costs. This final good is assembled by a representative final goods producer, that in turn requires differentiated inputs provided by a continuum of retailers. The latter set prices in a monopolistic competitive fashion subject to nominal rigidities and require intermediate goods to produce their output. These are provided by a set of competitive intermediate goods producers that require capital and labor services as inputs. The latter are an aggregate of different labor varieties, the wage for is decided by monopolistic competitive unions that are also subject to nominal rigidities. As Bayer et al. (2024), I make the simplifying assumption that firms solving forward-looking problems (such as the retailers' price setting problem) discount the future at the households' discount parameter β.[12]

4.2.1 Final goods production

The problem of the final goods producer is equivalent to the one described in Section 2.1.2 and thus omitted. However, I now allow the elasticity of substitution between different varieties to exogenously vary over time, i.e. I introduce what is commonly referred to as "cost-push" shocks in the literature. For notational convenience, I define $\mu_t := \frac{\epsilon_t}{\epsilon_t - 1}$ to denote the target mark-up of the retailers presented in the next section.

4.2.2 Retailers

There is a unit mass of retailers, each of which produce a given variety of the differentiated input as monopolist, taking into account demand schedule (4). Their only input are

[12]Since I will linearize the model with respect to aggregate shocks, only the steady-state value of the discount factor in the firms' dynamic problems will matter for the dynamic model responses. Bayer et al. (2019) and Lee (2021) report that using different specifications does not significantly affect results in their 2-asset HANK models with many similar features.

intermediate goods, which they purchase at real price mc_t (also referred to as "marginal costs") from the competitive intermediate goods producers. However, they are subject to nominal rigidities à la Calvo (1983) with price indexation, i.e. in any given period their nominal price remains fixed with probability λ_Y.

If receiving a re-set opportunity, a retailer will choose a price to maximize the corresponding expected net present value of real profits

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_Y^t \left(\frac{p_{jt}^*}{P_t} - mc_t \right) \left(\frac{p_{jt}^*}{P_t} \right)^{\frac{-\mu_t}{\mu_t - 1}} Y_t \quad .$$

Log-linearizing the first order conditions of the resulting price setting problem gives rise to the standard log-linear Phillips curve

$$\log(\pi_t) = \kappa_Y \left(mc_t - \frac{1}{\mu_t} \right) + \beta \mathbb{E}_t \log(\pi_{t+1}) \tag{24}$$

with $\kappa_Y := \frac{(1-\lambda_Y)(1-\lambda_Y\beta)}{\lambda_Y}$.

4.2.3 Intermediate goods producers

The homogeneous intermediate good is produced by a continuum of firms that use a constant-returns-to-scale technology represented by production function

$$F_t(u_t K_t, H_t) = Z_t F(u_t K_t, H_t) = Z_t (u_t K_t)^{\alpha} H_t^{1-\alpha} \quad . \tag{25}$$

K_t and H_t denote the input of capital and labor services. u_t is the degree of capital utilization that determines capital depreciation according to

$$\delta(u_t) = \delta_0 + \delta_1(u_t - 1) + \frac{\delta_2}{2}(u_t - 1)^2$$

and Z_t is a shock to Total Factor Productivity (TFP). Taking the price h_t for labor services as well as the capital rental rate r_t and its output price mc_t as given, an intermediate goods producer solves the static profit maximization problem

$$\max_{K_t, H_t, u_t} mc_t F_t(u_t K_t, H_t) - h_t H_t - (r_t + q_t \delta(u_t)) K_t \quad ,$$

the solution of which can be characterized using the following first order conditions:

$$h_t = (1-\alpha) mc_t Z_t (u_t K_t)^{\alpha} H_t^{-\alpha} \tag{26}$$

$$r_t + q_t \delta(u_t) = \alpha mc_t Z_t u_t (u_t K_t)^{\alpha-1} H_t^{1-\alpha} \tag{27}$$

$$q_t(\delta_1 + \delta_2(u_t - 1)) = \alpha mc_t Z_t (u_t K_t)^{\alpha-1} H_t^{1-\alpha} \quad . \tag{28}$$

4.2.4 Capital goods producer

Capital goods producers use the final good as input and operate a technology subject to adjustment costs: Using I_t units of the final good, they can produce

$$Z_t^I \left[1 - \frac{\phi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t$$

units of capital. Investment-specific productivity Z_t^I is exogenous and potentially following a time-varying shock process. Taking the price of capital q_t as given, the producers choose I_t to maximize the net present value of real profits

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(q_t Z_t^I \left[1 - \frac{\phi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t - I_t \right)$$

and their optimal interior solution will fulfill first-order condition

$$1 + q_t Z_t^I \left(\frac{\phi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - 1 + \phi \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right) = \beta q_{t+1} Z_{t+1}^I \phi \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 . \tag{29}$$

4.2.5 Labor market

The labor market follows a standard set-up as in Auclert et al. (forthcoming). Labor services are produced by a representative labor packer that aggregates a range of differentiated labor inputs $u \in (0, 1)$ according to

$$N_t = \left(\int_0^1 N_{ut}^{\frac{\epsilon_w - 1}{\epsilon_w}} du \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}$$

and will thus demand

$$N_{ut} = \left(\frac{W_{ut}}{W_t} \right)^{-\epsilon_w} N_t \tag{30}$$

of each labor variety. Each of the differentiated labor types is supplied by an union that sets the nominal wage W_{ut} as a monopolist to maximize the utility of its members, which are required to work according to a uniform schedule, i.e. all u workers have to supply the same amount of hours N_{ut}. Unfortunately, every period the leadership of a union u suffers utility costs $\frac{\psi}{2} \left(\frac{W_{ut}}{W_{ut-1}} - \pi_{SS} \right)^2$ for changing the nominal wage, perhaps due the administrative burden of adjusting contracts. In turn, every union solves

$$\max_{\{W_{u\tau}\}_{\tau=t}^{\infty}} \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left(\int \left(u(c_{i\tau}) - \varsigma \frac{N_{i\tau}^{1+\gamma}}{1+\gamma} \right) di - \frac{\psi}{2} \left(\frac{W_{ut}}{W_{ut-1}} - 1 \right)^2 \right) , \tag{31}$$

taking demand schedule (30) into account.

Households are exogenously distributed over unions in a uniform manner: Note that the law of large numbers applies thus also *within* unions so that the distribution of agents i overall and within any union u coincides.[13] Due to symmetry, the F.O.C.s of (31) then give rise to an aggregate *Wage Phillips curve* of the form

$$\pi_t^w (1 - \pi_t^w) = \kappa_w \left(\frac{\epsilon_w - 1}{\epsilon_w} (1 - \tau_p)(1 - \tau_t^w) \int \left(u'(c_{it}) \left(s_{it} \frac{W_t}{P_t} N_t \right)^{1-\tau_p} \right) di - \varsigma N_t^{1+\gamma} \right) + \beta \mathbb{E}_t \pi_{t+1}^w (1 - \pi_{t+1}^w) \tag{32}$$

[13]Since all labor varieties will be symmetric in equilibrium, her labor type u doesn't matter for an individual's consumption-saving problem.

with $w_t := W_t/P_T$ and $\pi_t^w := \frac{W_t}{W_{t-1}}$. For convenience, I define $\kappa_w := \frac{\epsilon_w}{\psi}$ to denote the slope of the Wage Phillips curve. Additional details on the derivation are provided in Appendix B.1.

4.3 Government

The government again consists of two branches, a monetary authority and a fiscal authority.

4.3.1 Monetary Authority

The monetary authority sets the nominal return $1 + r^R$ on a reserve asset that is in zero net supply. Specifically, it is assumed to follow a Taylor rule of the form

$$1 + r_{t+1}^R = \max\left\{(1 + r_{SS}^R)\left(\frac{1 + r_t^R}{1 + r_{SS}^R}\right)^{\rho^R}\left[\left(\frac{\pi_t}{\pi_{SS}}\right)^{\theta_\pi}\left(\frac{Y_t}{Y_{t-1}}\right)^{\theta_y}\right]^{1-\rho^R}\exp\left(\epsilon_t^R\right),\ 1 + r^{LB}\right\} \quad . \tag{33}$$

which features an Effective Lower Bound (ELB) denoted as r^{LB}. The parameter ρ^B introduces rate smoothing and if $\theta_y \neq 0$, the rule reacts to output growth in addition to inflation. ϵ_t^R represents an exogenous disturbance to the rule ("monetary policy shock"). Since the calibration will provide for a stable steady state, $1 + r_{SS}^R$ always constitutes the true long-run natural rate.

4.3.2 Fiscal Authority

The fiscal authority collects taxes, pays out transfers T_{it} and engages in government consumption G_t. It also issues nominal long-term government bonds which I introduce using a simple geometric maturity structure as in Bayer et al. (2023a). Bonds are long-lived. Every period, they pay one nominal unit of return and a random fraction δ^B of them retire without repaying the principal.[14] Denoting the nominal period t price of a bond as Q_t^B, its expected nominal return is given by $\mathbb{E}_t(Q_{t+1}^B(1 - \delta^B) + 1)$.

To state the government's budget constraint in a convenient form, let us define B_t^g to denote the value of public debt outstanding at the beginning of period t in terms of its period $t-1$ real market value Q_{t-1}/P_{t-1}. The dynamics of public debt must then be consistent with

$$B_{t+1}^g = (1 - \delta^B)\frac{Q_t^B B_t^g}{Q_{t-1}^B \pi_t} + G_t + \int_0^1 T_{ti} di + \frac{B_t^g}{Q_{t-1}^B \pi_t} - \Upsilon_t \quad , \tag{34}$$

[14]Equivalently, such a setting can be interpreted as featuring infinitely-lived bonds with geometrically declining coupon payments. See Woodford (2001).

i.e. the real period t market value of public debt outstanding at the end of period t equals the re-valued stock of public debt that did not retire plus the government's real spending obligations minus real tax revenues Υ_t. The latter equal

$$\Upsilon_t = \tau_t^{\Xi}\Pi_t + \int_{m^{\Xi}}^{1} \left(w_t s_{it} N_t - (1-\tau_t^w)\,(w_t s_{it} N_t)^{1-\tau^p} \right) di \quad .$$

As baseline, I assume both government spending items G_t and T_{it} to be solely determined by exogenous shocks. Without any occurring, they remain fixed at $G_t = G_{SS}$ and $T_{it} = 0$. To ensure public debt stability in the face of various business cycle shocks, the fiscal authority is furthermore assumed to adjust taxes as $\tau_t^w = \tau_t \tau_{ss}^w$ and $\tau_t^{\Xi} = \tau_t \tau_{ss}^{\Xi}$. The tax level τ_t evolves according to

$$\left(\frac{\tau_t}{\tau_{ss}}\right) = \left(\frac{\tau_{t-1}}{\tau_{ss}}\right)^{\rho_\tau} \left(\frac{B_t^g}{B_{ss}^g}\right)^{(1-\rho_\tau)\psi_B} \quad , \tag{35}$$

a functional form also used in Bianchi et al. (2023). Adjusting all tax levels proportionally by the same factor aims to reduce the distributional impact of fiscal consolidation in order to better isolate the role of the public debt level. Otherwise, the fiscal authority issues any amount of bonds B_{t+1}^g necessary to fulfill its budget constraint (34). Intuitively, policy rule (35) means that the government will eventually raise taxes to pay back debt in surplus of its long-run target, but may do so only slowly. In Section 8, I will consider the alternative scenario in which the fiscal authority consolidates its budget by adjusting spending G instead of taxes τ_t.

4.4 Liquid Asset Provision

While I assume a centralized market for claims to (illiquid) capital, households obtain liquid assets from a set of competitive *liquid asset funds* (LAFs). In contrast to households, these funds are able to trade claims to capital every period and also have access to a technology to short-sell any asset. Their objective is to maximize expected real returns by investing the funds A_{t+1}^l they receive from the households in capital, government bonds and reserves. In particular, the LAFs solve

$$\max_{B_{t+1}^l, R_{t+1}^l} \left\{ \mathbb{E}_t \left[(r_{t+1}^k + q_{t+1}) \frac{A_{t+1}^l - B_t^l - R_t^l}{q_t} + \frac{Q_{t+1}^B(1-\delta^B)+1}{\pi_{t+1}} \frac{B_{t+1}^l}{Q_t^B} + \frac{1+r_{t+1}^R}{\pi_{t+1}} R_{t+1}^l \right] \right.$$
$$\left. - A_{t+1}^l \left(\varphi + \frac{\Psi}{2} \left(1 - \frac{B_{t+1}^l + R_{t+1}^l}{A_{t+1}^l} \right)^2 \right) \right\} \quad , \tag{36}$$

where A_{t+1}^l denotes the total amount of assets intermediated by the LAF and B_t^l and R_t^l the amount of government debt and reserves it chooses to acquire. A fund faces costs for each unit of liquid asset it invests on behalf of the households. This involves a linear component φ and a part $\frac{\Psi}{2}\left(1 - \frac{B_t^l + R_t^l}{A_t^l}\right)^2$ that increases in the relative amount of the fund's asset positions that are *not* in liquid government assets. This structure implies that the equilibrium government bond prices Q^B must fulfill the no-arbitrage condition

$$\mathbb{E}_t \frac{1 + r^R_{t+1}}{\pi_{t+1}} = \mathbb{E}_t \frac{Q_{t+1}(1-\delta^B)+1}{\pi_{t+1} Q^B_t} \quad .$$

Furthermore, the LAFs' aggregate portfolio choice can be determined from the corresponding F.O.C.

$$\mathbb{E}_t \left(\frac{r^k_{t+1} + q_{t+1}}{q_t} \right) - \Psi \left(1 - \frac{B^l_{t+1}}{A^l_{t+1}} \right) = \mathbb{E}_t \left(\frac{1 + r^R_{t+1}}{\pi_{t+1}} \right) \tag{37}$$

and the ex-post real return to household's liquid savings will be given by

$$1 + r^l_t = \frac{q_t + r^k_t}{q_{t-1}} \frac{A^l_t - B^l_t}{A^l_t} + \frac{Q^B_t(1-\delta^B)+1}{\pi_t Q^B_{t-1}} \frac{B^l_t}{A^l_t} - \varphi - \frac{\Psi}{2} \left(1 - \frac{B^l_t}{A^l_t} \right)^2 \tag{38}$$

(anticipating that in equilibrium $R^l_t = 0$ as reserves are in 0 net supply).

A few words on the above assumptions are in order: The aim of the perhaps peculiar cost structure in (36) is *not* to provide a particularly realistic model of financial intermediation, but rather to introduce a parsimonious way to flexibly move between various assumptions on liquid asset supply in the literature. For this purpose, the parameter Ψ has a simple interpretation as determining how easily capital assets can be used for liquidity provision: In case $\Psi \to \infty$, the model nests the assumption of *segmented* markets for liquid and illiquid assets as in Kaplan et al. (2018) or Bayer et al. (2024), who assume that government bonds can *only* be held as a liquid asset and capital *only* as a illiquid asset, i.e. that capital is useless for the provision of liquid assets. In contrast, for $\Psi \to 0$ it nests a completely *integrated* market as in Auclert et al. (forthcoming), who assume that both capital and public debt can be held in either liquid or illiquid form. If that is the case, capital is a perfect substitute for government bonds for the purpose of liquidity provision and (37) collapses to a standard no-arbitrage condition. As will become clear below, being able to move between either extreme will be crucial for the model results.

While a micro-founded models of financial intermediation as e.g. in Chiang and Żoch (2023) could also provide for imperfect usefulness of capital for liquidity provision, the above formulation has several benefits: Most importantly, if the model's Steady State (SS) is calibrated so that the household sector's net liquid asset holdings equal the net supply of government bonds, one can move "in between" the above-mentioned assumptions on asset market structure *without* changing its SS. In contrast, with explicit models of financial intermediation as typically used in the DSGE literature (e.g. Gertler and Karadi, 2011), this would in general not be possible in the context of a HANK model, as allowing for varying degrees of liquidity transformation typically affects the steady state by requiring different financial sector net worth or similar. Besides, my simple structure makes it particularly transparent how varying the usefulness of capital for liquidity provision is achieved.

4.5 Market clearing conditions and equilibrium

The Definition of Equilibrium is standard, but tedious, given that the quantitative model features multiple markets and also requires keeping track of the evolution of the aggregate distribution. In turn, I relegate these details to Appendix B.2.

4.6 Numerical Approach

To approximate the dynamic equilibrium of the model, I use established techniques that conduct first-order perturbation around the economy's non-stochastic steady state, specifically the Sequence Space Jacobian (SSJ)-method proposed by Auclert et al. (2021) and the State Space method employed by Bayer et al. (2024). Both have comparative advantages for different purposes: For example, the Sequence Space method can more conveniently handle a binding ELB relevant for the analysis in Section 9, as one can more easily impose the lower bound on nominal rates via monetary news shocks as in McKay and Wieland (2021). As we see below, representing the linearized model in terms of Impulse Response Functions (IRFs) will also be useful to isolate the "debt inflation" in that Section's complex scenario. In contrast, a State Space method allows to easily check whether the model admits a unique and stable solution for a given parameterization via the Blanchard-Kahn-conditions and can provide a comparison to evaluate on whether the truncation horizon required for the Auclert et al. (2021) method is long enough.
For obtaining the model's steady state, I use a multidimensional Endogenous Grid Method similar to the algorithm described in Bayer et al. (2019) to solve the households' dynamic programming problem. The joint income- and asset distribution is approximated as a histogram using the "lottery"-method proposed by Young (2010). Further details on the numerical implementation are provided in Appendix B.3.

5 Calibration of the quantitative model

A period is interpreted to be a quarter. I aim for the model to be consistent with the most relevant features of the US economy, including its income- and wealth distribution, as well as the key empirical moments emphasized by the HANK literature, in particular the presence of poor and wealthy "Hand-to-Mouth" (HtM) households in the economy and a fairly high aggregate MPC. To do so, I first set a range of parameters exogenously, relying on the previous literature: In addition to standard preference- and technology parameters, this includes some parameters exclusively affecting the dynamic model response to aggregate shocks, for which I rely on previous papers estimating a HANK model. Afterwards, the remaining parameter values are chosen to match several steady state moments of the household wealth distribution.

5.1 Externally calibrated

The model's externally calibrated parameters are displayed in Table 1: I set the households' risk aversion parameter to $\xi = 1.5$, within the range of standard values used in the literature. Regarding technology, I use the standard values of $\alpha = 0.33$ for the Cobb-Douglas parameter for capital and set a quarterly depreciation rate for capital of $\delta = 0.0175$, implying approx. 7% annual depreciation. Similar, I set μ_t to a conventional value of 1.1, resulting in a steady state markup of 10%. The elasticity of substitutions between different labor varieties is assumed to be the same as for goods and thus $\epsilon_w = 11$.[15] The slope of the price Phillips curve is set to $\kappa_Y = 0.06$, in line with the recent evidence by Gagliardone et al. (2023). In the HANK literature, wages are often assumed to be substantially stickier than prices, even though estimated DSGE models do not always support this. I set $\kappa_w = 0.015$ to be consistent with the former, a value based on the estimate of Auclert et al. (2020). However, given the empirical controversies surrounding these parameters, related robustness checks will be discussed in Section 8.

Several other parameters governing the economy are calibrated following Bayer et al. (2024): First, I also set the probability of exiting the $\Xi = 1$ state within a given period to be 6.25% and adopt their tax progressivity parameter $\tau_p = 0.12$. The investment adjustment cost is chosen to be 3.5 and the ratio δ_2/δ_1 set to be 1, reflecting the results of their model estimation. For a given δ_2/δ_1-ratio, I always set δ_1 and δ_2 to achieve $u_t = 1.0$ in steady state. Again, since estimates for the investment- and utilization adjustment parameters also vary in the literature, these values will be subjected to robustness tests as well.

Regarding monetary policy, I parameterize the model's interest rule with standard values also employed by Bayer et al. (2023b) to study post-2020 macroeconomic dynamics in the US. In particular, this includes setting the Central Bank's inflation reaction parameter to the value $\theta_\pi = 1.5$ first proposed by Taylor (1993), making it clear that monetary policy is "active" in my model. The ELB is set to be 2 percentage points (in annual terms) below the steady state policy rate r^R_{SS}, given that my exercise in Section 9 effectively assumes the model to be in steady state before 2020. In that year, the nominal rate was around 2% and the ELB thus 2 p.p. below it.

On the fiscal side, the steady labor tax level τ is chosen so as to be consistent with $G/Y \approx 17.5\%$ given the model's other targets, in line with the average ratio of government consumption and investment to GDP in 2014-2019. The tax rate on the "entrepreneurs"'s profit incomes are set to 24%, reflecting the top tax bracket for qualified dividends in the US. Bianchi et al. (2023) report the average maturity of US treasury debt to typically vary between 4.5-5.5 years, so I chose $\delta^B = 0.05$ to be consistent with an average 5-year (20 quarter) duration. While the same authors estimate their tax rules to be very persistent and react only very little to public debt, their values wouldn't necessarily result in a stable

[15] Since I set κ_w independently of ϵ_w and calibrate ς to achieve $N_{ss} = 1$, the value of this parameter is practically of limited importance.

Parameter	Description	Value	Source
ξ	Risk aversion	1.5	Standard
ι	Exit prob. entrepreneurs	1/16	Bayer et al. (2024)
α	Cobb-Douglas parameter	0.33	Standard
δ_0	Steady State depreciation	0.0175	Standard
μ	SS Goods markup	1.1	Standard
κ_Y	Slope of price Phillips curve	0.06	Gagliardone et al. (2023)
κ_w	Slope of wage Phillips curve	0.015	Auclert et al. (2020)
ϵ_w	EOS labor varieties	11	Standard
ϕ	investment adjustment cost	3.5	Bayer et al. (2024)
δ_2/δ_1	utilization parameters	1.0	Bayer et al. (2024)
γ	Inverse Frisch	1.0	Standard
δ^B	Government debt duration	0.05	5 years avg. maturity
τ	Tax level	0.2	$G/Y \approx 17.5\%$
τ^p	Tax progressivity	0.12	Bayer et al. (2024)
τ^Ξ	Profit Tax	0.24	US Tax Code
$(\rho_R, \theta_\pi, \theta_y)$	Taylor rule parameters	(0.8,1.5,0.2)	Bayer et al. (2023b)
R^{LB}	Effective Lower Bound	$r^R_{SS} - 0.005$	2 p.p. below r^R_{SS}
(ρ_τ, ψ_B)	Tax rule parameters	(0.94, 0.5)	See text

Table 1: Externally set parameters

equilibrium in a HANK model with real interest rates depending on the public debt level. In turn, I set $\rho_\tau = 0.94$ and $\psi_B = 0.5$, ad hoc values that result in a reasonably drawn-out public debt responses for the exercise in Section 9. In particular, these parameters imply that for a simple transfer shock as studied in Section 7, public debt will start to decline only after approx. 4.5 years and require roughly 20 years to return to its stationary level. Further robustness checks reveal that as long as they induce persistent deviations of public debt, results will be robust to different values.

5.2 Internal calibration

The remaining parameters are chosen so that the model matches various targets in the non-stochastic steady state. To clarify how they come about, I present for each parameter the moment I use to identify it. While in principle any parameter will somewhat affect any of the stationary equilibrium's target moments, it nevertheless turns out that achieving a good fit with the target relies mostly on the stated parameter.

Several parameters are disciplined by moments related to the steady state wealth distribution. I choose the household discount factor β to match a ratio of average steady state capital holdings to output of 11.22 as in Bayer et al. (2024), resulting in $\beta = 0.9838$. The probability ζ determines the amount of "super rich" entrepreneur households and I use it to target a Top 10% wealth share of 70%, approximately the value computed by Krueger

Parameter	Description	Value	Target
β	Time discounting	0.9838	$K/Y = 11.44$
ζ	prob. entrepreneur state	0.0005	Wealth share top 10
λ	prob. illiquid asset adjustment	0.0363	$B/Y = 1.8$
$\bar{R}$	Borrowing penalty	0.0355	16% borrower share
$\underline{a}$	Borrowing limit	-1.4491	100 % avg. quart. income
G_{ss}	Government consumption	0.5649	Budget clearing (34)
φ	Liquidity Fixed Cost	0.0092	$r^l_{ss} = 0.0$
Ψ	Liquidity Supply	0.005	See Section 6

Table 2: Internally calibrated parameters

et al. (2016) using SCF data. This requires a value of approx. 0.0005.
λ determines the (il-)liquidity of capital and thus how many liquid assets agents wish to additionally hold for self-insurance purposes: I use it to target net liquid asset holdings by households to equal 1.8 times quarterly GDP. Firstly, it is in line with the amount of *domestically* held public debt in the US before the start of the 2020 Covid pandemic, the arguably relevant measure in my closed-economy model. However, the target is also close the average *overall* debt-to-GDP ratio for the US over the period 1970-2019 and can thus also be interpreted in this way.[16] Additionally, it is also roughly in line with the average net amounts of liquid assets held by US households (HHs) over the 2014-2019 period.[17]
Regarding household borrowing, I follow Kaplan et al. (2018) by assuming the borrowing limit to equal the average quarterly (post-tax) labor income and set the borrowing penalty $\bar{R}$ so as end up with 16% of households having $a \leq 0$ in SS. The return wedge φ is set so that the real return to liquid savings is 0 in SS as in Bayer et al. (2023b), which requires setting φ equal to the steady state return on capital, equal to 0.0092 in the Baseline model (a return of 3.7% in annual terms). Section 6 below elaborates on choice of the parameter Ψ.

5.3 Distributional Moments

In this section, I validate the internal calibration by analyzing various model-generated moments that were not directly targeted.
Table 3 compares various untargeted moments of the model's Steady State income- and

[16] The statement regarding *domestically-held debt* is based on subtracting FRED series FDHBFIN (Federal Debt held by foreign and international investors) from FYGFDPUN (Federal Debt Held by the Public), while the overall debt-to-GDP ratio is taken from FYGFGDQ188S.

[17] For the purpose of this calculation, I define net liquid asset holdings as the sum of cash and checkable deposits, money market funds and direct treasury security holdings minus revolving consumer credit and credit card debt. According to the Federal Reserve's Financial Accounts of the United States, on average the holdings of HHs and Non-Profit Organizations (NPOs) equalled 54% of quarterly GDP over that period. Since NPO assets are not necessarily liquid from the point of view of HHs, holdings of the latter should be somewhat lower.

	Disposable Income		Net Worth	
	Model	Data	Model	Data
Quint. 1	6.8	4.5	0.0	-0.2
Quint. 2	10.8	9.9	1.2	1.2
Quint. 3	14.8	15.3	4.2	4.6
Quint. 4	20.6	22.8	11.0	11.9
Quint. 5	46.9	47.5	83.8	82.5
Gini	0.40	0.42	0.80	0.78

Note: "Data" refers to moments computed by Krueger et al. (2016) using PSID and SCF.

Table 3: Distributional moments comparison

wealth distributions with their empirical counterparts as reported by Krueger et al. (2016). The latter are based on the 2006 Panel Survey of Income Dynamics (PSID) and the 2007 Survey of Consumer Finance (SCF), respectively.[18] Arguably, the model achieves a fairly good fit, in particular for Net Worth.

Since I am employing a two-asset model, it is not only relevant to assess how closely the framework matches data moments related to the distribution of overall net worth, but also the different asset classes held by the households. I do so in Table 4: First, I am considering moments of the illiquid- and liquid wealth distribution separately. In particular, I compare them with statistics reported by Kaplan et al. (2018), who rely on the 2004 SCF. As in the data, the model generates a more unequal distribution of liquid assets and ownership of both asset classes is concentrated in their respective Top 10%, with the bottom 50% holding hardly any. Also, the model moments of the illiquid asset distribution are close to the data, mildly under-predicting the share of the Top 10%. However, for liquid assets, I generate a comparably more equal asset distributions, with the share held by the Top 10% not as high and the share of the Next 40% substantially larger than in the SCF data. But, as noted by Kaplan et al. (2018), it is "notoriously challenging" to match the extreme right tail of wealth distributions with income risk alone. From that perspective, I view my model's performance as satisfactory.

Finally, I analyze how many households are *Hand-to-Mouth* (HtM) in the sense of Kaplan et al. (2014), i.e., whether their liquid asset holdings are less than 2 weeks ($\approx 1/6$ of a model period) of current household income above of either 0 or the borrowing constraint. I also classify them as "Wealthy HtM" if they additionally hold illiquid assets and "Poor HtM" if they do not. The model matches the empirical evidence on the size of either group of agents well. As visualized in Figure 2, these low liquid-wealth agents tend to have particularly high MPCs. In turn, my framework is able to generate an average

[18] In the data, disposable income is defined as the sum of after-tax earnings, income generated by assets held as well as unemployment benefits. In the model, it only compromises the first two as there is unemployment. In both model and data, Net Worth relates to both liquid and illiquid assets.

Moments	Model	Data (incl. source)
Illiquid asset shares		(from Kaplan et al., 2018)
Top 10%	67.1	70
Next 40%	31.4	27
Bottom 50%	1.5	3
Liquid asset shares		(from Kaplan et al., 2018)
Top 10%	74.7	86
Next 40%	24.6	18
Bottom 50%	0.7	-4
Hand-to-Mouth (HtM) Status		(from Kaplan et al., 2014)
Share HtM	29.0	31.2
Share Wealthy HtM	17.6	19.2
Share Poor HtM	11.4	12.1

Table 4: Portfolio moments: Model vs. Data

quarterly MPC of 15.8% and an average annualized MPC of 36.7%.[19] The former is of a similar magnitude as the corresponding value reported by Kaplan et al. (2018).

6 Liquidity and real rates

The "debt inflation" mechanism explained in Section 2 depended on higher public exerting upward pressure on the real (liquid asset) interest rate. Thus, to assess related model results, it is important to consider whether such effects are of a magnitude that can be deemed "reasonable" one way or another. A natural candidate to do so is checking consistency with the empirical evidence summarized in Section 3.1, i.e., that in the medium- to long run a 1 percentage point (p.p.) increase of the economy's annual debt-to-gdp ratio is associated with an increase of annual real treasury returns of roughly 3-6 bp.

6.1 Liquidity and real rates in the 2-asset HANK model

Can the 2-asset HANK model generate a relationship of this magnitude under the asset market structures used in the previous literature, i.e., either $\Psi \to \infty$ or $\Psi \to 0$? Computing new steady states for different Debt-to-GDP ratios under the parameterization specified in Section 5 yields Figure 3.[20] The results are rather stark: Under segmented asset markets, a 1 p.p. higher annual Debt-to-GDP ratio causes the annual steady state

[19]I compute individuals' annualized MPCs $aMPC$ as $aMPC = 1-(1-qMPC)^4$ following Carroll et al. (2017). Note that these *annualized* MPCs will not exactly equal individuals' *annual* MPCs.

[20]For this exercise, the following assumptions on government policy are made: The central bank adjusts its nominal rate target so that $\pi_{ss} = 1$ is also achieved in the new steady state. At the same time, the fiscal authority keeps G_{ss} at the same value as in the Baseline SS and adapts the tax level τ_t to clear its

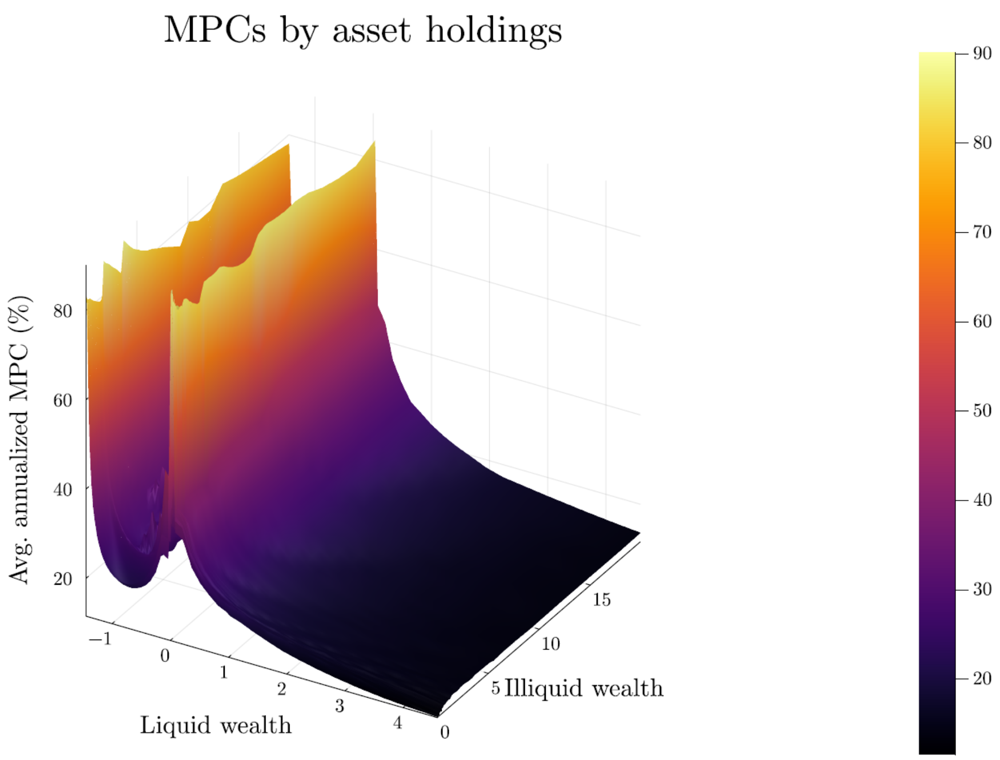
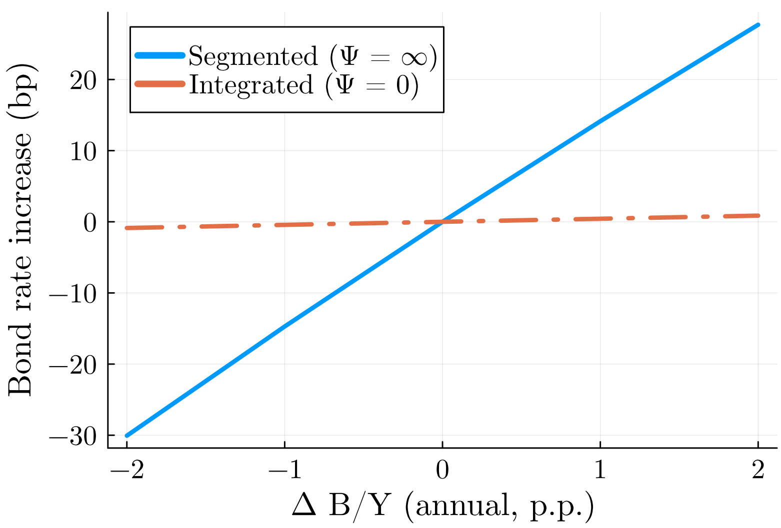


Figure 2: Model MPC distribution

Figure 3: Model: Long-run effects of gov't debt on bond returns



Note: $\Delta B/Y$ denotes the change in the gov't debt-to-GDP ratio compared to the Baseline Steady State.

real treasury return $1/Q^B + (1 - \delta^B)$ to increase by approx. 15 bp, almost 3 times *more* than the upper end of the empirical estimates. In case of the integrated asset market ($\Psi \to 0$), we have the polar opposite: The response of the real liquid rate is much *smaller* and hardly noticeable, not even a third of the empirical estimates' lower range.

Is this (perhaps surprising) result simply a peculiarity of the model proposed in this paper or a more general feature of 2-asset HANK models? Investigating the sources of this result suggests the latter: As is well known, such frameworks need to feature a sufficiently high gap between the return on liquid and illiquid assets to give rise to relatively high MPCs (c.f. Kaplan and Violante, 2022). With that, the model can generate a substantial number of Wealthy HtM agents as households are incentivized to forego holding large amounts of liquidity in order to reap the illiquid assets' higher returns. In that case, however, it also seems intuitive that if agents are to hold more liquid government bonds, they will have to be compensated with substantially higher returns. This argument, in turn, predicts the interest rate effects of public debt in 2-asset HANK models with *segmented* asset markets to be closely linked to their initial return gap and MPCs. On the other hand, with *integrated* markets, public debt can just crowd out a bit of the much larger capital stock and does not necessarily require households to hold substantially more liquid assets. Therefore, there is no reason to expect a tight connection.

To analyze the link between the initial return gap and interest rate effects of public debt supply in the segmented $\Psi = \infty$ case more formally, I re-calibrate my baseline framework to provide for lower steady state return gaps. In particular, I aim for the new parameterization to remain consistent with the aggregate moments targeted in 5.2 and just increase the long-run rate of capital depreciation δ_0: Keeping all other externally-set parameters the same and matching the same targets, increasing δ_0 in several steps from 0.0175 to 0.025 (still a standard calibration choice) will yield substantially lower capital returns and thus rate gaps in the resulting stationary equilibria.[21] A summary of the respective parameter results is provided in Appendix Table D.1: Overall, if the model is to remain compatible with the same moments under a lower returns gap, it requires a higher λ (i.e. capital to be less illiquid) and households to be more patient (higher β). The former is because the lower rate gap makes capital relatively less attractive, so it needs to be less illiquid for the aggregate household portfolio to remain the same. In turn, the latter is necessary as a higher δ_0 decreases the returns of capital. This obliges households to be more patient if they are to still hold the same amount.

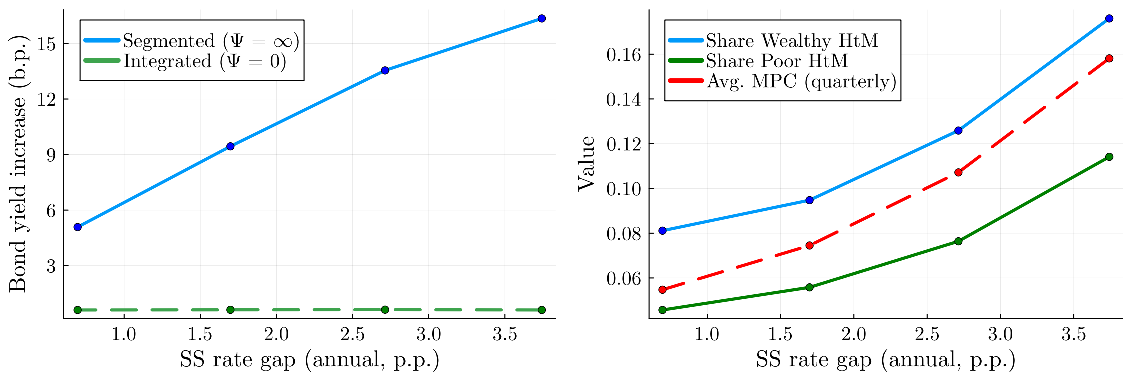
Figure 4 then visualizes the implications of different initial return gaps: In Panel 4a, we see that calibrating the model to be consistent with a lower initial return gap indeed decreases the elasticity of government bond returns with respect to bond supply in the case of segmented asset market. In fact, it can even generate a response within the 3-6 b.p. range if the return gap is low enough. But as Panel 4b illustrates, this will render the

budget.

[21] This exploits the close connection between capital returns and the (targeted) K/Y-ratio in Cobb-Douglas production functions: In steady state, (27) requires $r_t^k = \alpha \frac{1}{\mu_{ss}} \frac{Y}{K} - \delta_0$.

Figure 4: Implications of steady state return gaps

(a) Liquid rate elasticity (b) Micro-moments



Note: "Bond Yield Increase" refers to the difference of the annualized bond yield $\frac{1}{Q^b} + (1 - \delta^B)$ from the calibrated Steady State after solving for a new stationary equilibrium with 1 p.p. higher annual Debt-to-GDP ratio. "SS rate gap" denotes the difference between annualized r^k and r^l in the calibrated steady state.

model unable to generate high average MPCs and substantial amounts of HtM households. In line with the arguments above, under *integrated* asset markets, the response of interest rates to public debt supply doesn't change much with the calibration and is always low. As already noted in Section 1.1, the above results suggest a tension in the HANK literature on fiscal policy: While it argued both high MPCs and their liquidity supply effects to be important for the aggregate effects of fiscal policy, the discussed modelling assumptions prevent both effects playing a role at the same time. [22]

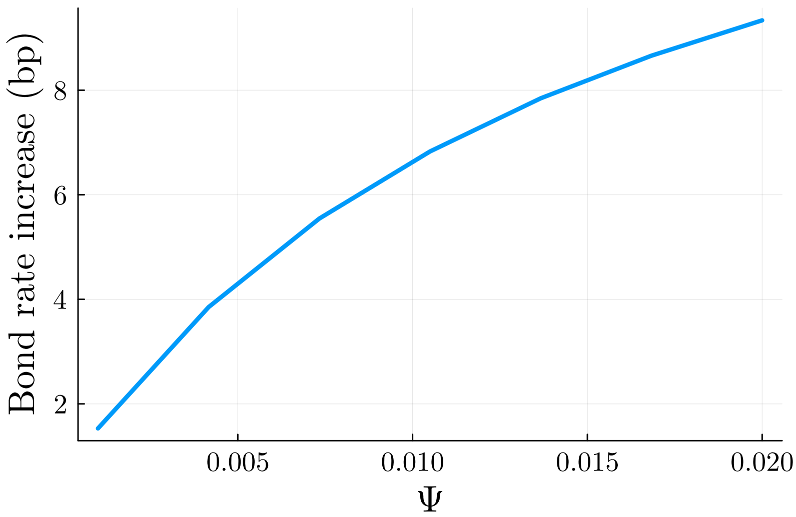
6.2 Resolution: Asset market structure

Overall, the above results for the off-the-shelf assumptions on asset market structure are somewhat unsatisfying, not only because they fail to generate certain results but also since they indicate a drawback of heterogeneous agent business cycle theory: While related frameworks can relate to rich cross-sectional evidence, they may also require subtle modelling choices that matter for aggregate outcomes but cannot be disciplined by micro-data alone.

Nevertheless, casting the model's asset market structure in terms a single parameter Ψ immediately suggests a resolution for the particular issue at hand: If effects are overly strong for $\Psi \to \infty$ and overly weak for $\Psi \to 0$, then a value in between will presumably result in a reasonable magnitude. To explore this possibility, Figure 5 displays, for different values

[22] While e.g. Bayer et al. (2023a) report their framework to generate a long-run response of liquid rates to public debt supply in line with Rachel and Summers (2019), their calibration seems to imply a rate gap of only 1.5% annually as well as an average quarterly MPC of less than 5%, i.e. their framework also seems to be subject to the calibration trade-off.

Figure 5: Treasury return responses by Ψ



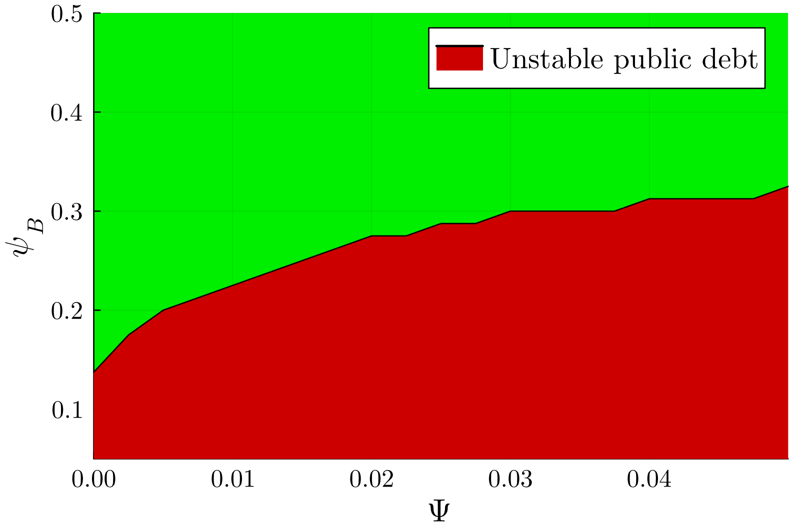
Note: "Bond rate increase" refers to the difference of the annualized bond yield $\frac{1}{Q^b} + (1 - \delta^B)$ from the calibrated Steady State after solving for a new stationary equilibrium with 1 p.p. higher annual Debt-to-GDP ratio.

of Ψ, how much the steady state return of treasury securities changes after a long-run 1 p.p. increase in the Debt-to-GDP ratio. We see that for values between 0.003 and 0.0075, the liquid return reaction in the model economy is indeed in line with the range proposed by Rachel and Summers (2019). As baseline calibration, I will adopt $\Psi = 0.005$, which results in a response close to Laubach (2009)'s estimate of 4 basis points.

It is worth noting that considering the impact of the asset market structure in 2-asset HANK models is important beyond this paper's immediate concern with inflation. Amongst others, the effect of public debt supply on the interest rates a government faces is a key variable for public debt sustainability, a topic also analyzed using HANK models (e.g. Langot et al., 2024). To illustrate this point, Figure 6 indicates which combinations of Ψ and the fiscal policy parameter ψ_B result in a locally stable macroeconomic equilibrium, keeping all other parameters as specified in Section 5. Clearly, for higher values of Ψ, fiscal policy needs to react stronger to the stock of public debt (higher ψ_B) to ensure stability of the policy regime.

Furthermore, as we will see clear below, the asset market structure also matters for the dynamics of investment and the capital stock. Here, an interesting aspect of the 2-asset model is that its high-Ψ variants with stronger reaction of liquid returns provide for *less* crowd-out of private capital in response to government debt. In contrast, in standard incomplete markets models with just a single asset market, the effect on interest rates is usually stronger if it crowds out *more* capital.

Figure 6: Model Stability for different parameters



Note: The figure indicates whether the HANK model has a unique stable equilibrium for different combinations of the parameters ψ_B and Ψ, respectively. For combinations in the red-shaded area, it does not.

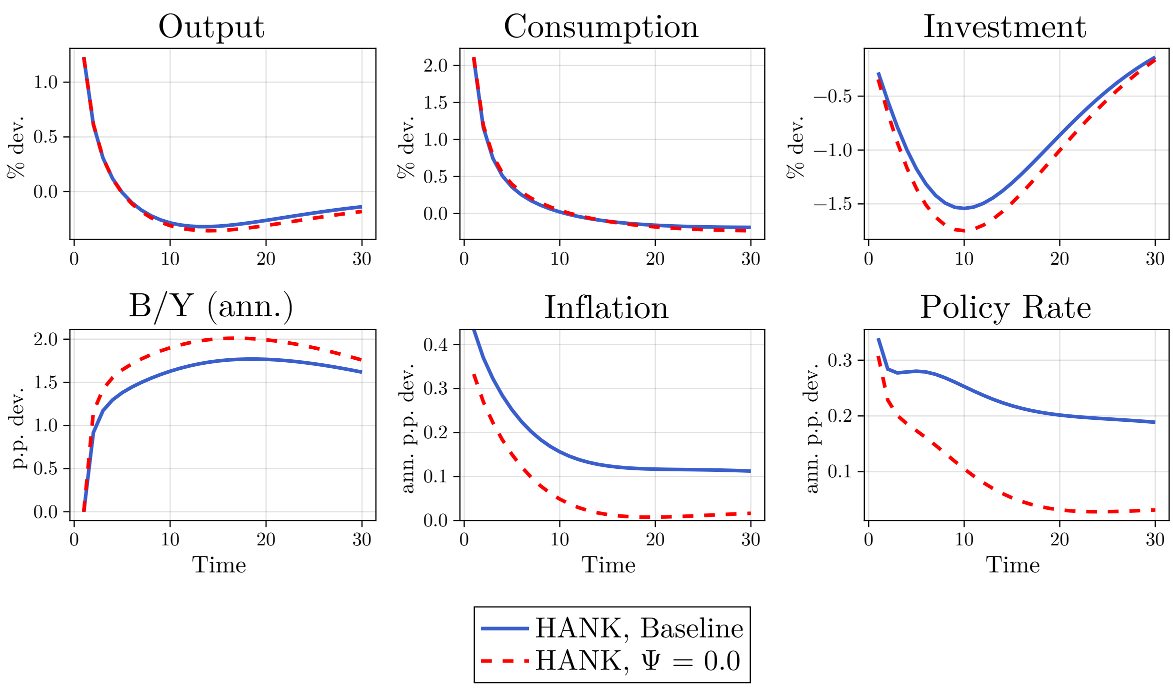
7 Debt-driven inflation in HANK

Armed with the calibrated 2-asset HANK model, we are now ready to assess whether public debt can indeed induce relevant amounts of inflation through its effect on the natural rate. To keep the analysis in this section as transparent as possible, I do so in a particular simple scenario providing for a substantial reaction of public debt: The only exogenous disturbance will be a one time-shock to government transfers T_t without any persistence, which may be viewed as the government sending out "stimulus checks". To aid this interpretation, I choose the size of the shock to amount to 2% of Steady State GDP. In terms of the USA's 2019 GDP, this would amount to circa USD 1,300 per capita, roughly the size of the one-time payments distributed under the CARES act in 2020.

7.1 Isolating the Debt-driven Inflation

Pinning down the liquidity-driven inflation in general equilibrium is a non-trivial undertaking, since many other features of the HANK model will also affect the response of inflation to the fiscal shock: Relating to the TANK example invoked earlier, the presence of constrained household with high MPCs by itself can induce a substantial inflation response to a transfer shock even without any specific usefulness of public debt as means of insurance. However, the structure for liquid asset provision presented in Section 4.4 offers a resolution: Changing the parameter Ψ allows me to vary the potential effects of public debt supply on the natural rate while keeping everything else, in particular the steady state income- and wealth distribution, the same. In particular, the difference between my baseline model and a $\Psi = 0$ version that provides for only a very small real interest rate

Figure 7: Model IRFs to transfer shock



Note: B/Y represents the real market value of public debt B^g over annualized GDP. Figures display relative (in %) or percentage point (p.p.) deviations from Steady State. An expanded figure with more IRFs is provided as Figure E.1 in Appendix E.

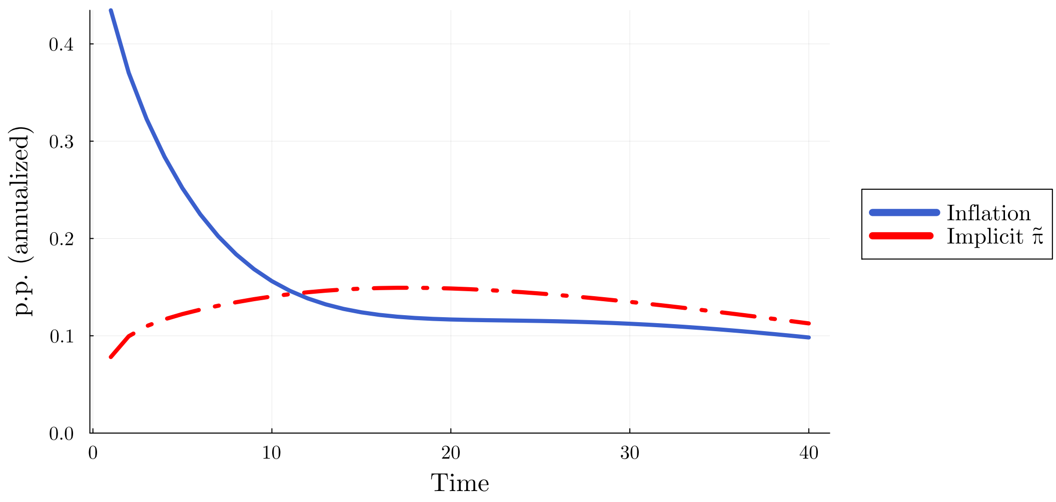
impact of debt supply can isolate its General Equilibrium inflationary effects through the "natural" rate.

Figure 7 displays the model results to the fiscal "stimulus" shock. The response of consumption looks as we would expect in a model featuring a high of amount of HtM-households with high MPCs, increasing substantially at impact and fading out after a few quarters. Output and inflation similarly increase at impact, causing the Central Bank to raise its nominal rate. At the same time, investment decreases, which reflects crowd-out amid higher real interest rates after the shock. This, in combination with slowly increasing taxes to combat the now-higher public debt, causes the output- and consumption-responses to turn negative after a few quarters.

Comparing the Baseline model (blue line) to its counterpart with integrated asset markets ($\Psi = 0$, red-dashed line), we see that public debt indeed has the inflationary impact predicted by the simple model: If it has a particular advantage to serve as a means of consumption smoothing, inflation rises more on impact and remains elevated while public debt is as well. This happens despite the Central Bank keeping the interest rate higher for longer in the baseline model, and, in line with the insights from the analytical model, after the redistributive fiscal shock has ceased.[23] While the overall magnitude of the effect appears moderate, it is of a magnitude policymakers should arguably be concerned about,

[23] While every household receives the same amount under the transfer policy, it can be seen as redistributive in the sense that poor household's receive a higher amount relative to their available resources.

Figure 8: Transfer shock: Model inflation vs. $\tilde{\pi}$



particular if a fiscal expansion raises the value of debt beyond the one generated in this simple scenario.

As one would expect, allowing public debt to crowd out capital at no cost in the setting with integrated asset markets results in a somewhat stronger investment decline, but remarkably, hardly affects the output- and consumption responses to the shock. It may initially seem unintuitive that the market value of public debt rises more strongly with $\Psi = 0$, as in this case the fiscal authority's refinancing cost do increase less after the shock. However, it can be explained by smaller initial valuation losses due to the smaller nominal rate reaction compared to the Baseline economy.

Summing up, we see that natural rate pressure stemming from government bonds' time-varying rate premium as means of self-insurance appears to have the expected effects on inflation in the quantitative HANK model and is arguably quantitatively relevant. However, investment demand, a margin not featured in the analytical model, appears to be an important component of the underlying demand pressure, an insight that will be elaborated in the next section.

An interesting side question is whether public debt's inflationary effect aligns with the magnitudes predicted by simple formulas such as (15)? This may also serve as an additional check on this section's results. I hence use the Baseline model's long-run interest rate response to the public debt-to-GDP rate to approximate a "debt-corrected" natural rate $\tilde{R}_t$ as

$$\log \tilde{R}_t = \log(1 + r_{ss}^R) + \theta^B \left(\log(B_{t+1}^g / Y_t) - \log(B_{ss}^g / Y_{ss})\right) \quad . \tag{39}$$

Here, θ^B denotes the approximate steady state elasticity of the gross liquid return with respect to the quarterly Debt-to-GDP ratio, which amounts to 0.0047 in my baseline calibration. Using that value, I compute $\tilde{\pi}$ as in (15) and compare it with the model-implied time path of inflation in Figure 8. The magnitudes seem approximately in line, but we note the "implicit target"-formula to over-predict medium-term inflation in the aftermath of the shock. Of course, since $\tilde{\pi}$ is based on a formula not accounting for nominal rate persistence, an output reaction or general equilibrium effects, one shouldn't expect them to correspond perfectly. Nevertheless, it seems that simple formulas as the one for $\tilde{\pi}$ are indeed useful for getting a grasp of the "debt inflation".

Before continuing, I also wish to stress that just because the differences between the model responses in Figure 7 may not seem that large, it does not mean that the asset market structure's aggregate impact is limited. To illustrate that point, Appendix Figure E.2 additionally compares the responses with the case $\Psi = 0.05$, an arbitrary high value implementing an outcome close to the "segmented" economy. Compared to the Baseline, one can see big differences for both inflation and real outcomes like investment and the value of public debt. Interestingly, the consumption response to the shock is almost the same in all three cases. This indicates that in complex HANK models, asset market arrangements and investment may shape inflation more than realized consumption behavior. Overall, the take-away from this additional comparison is that both for studying the inflationary and real effects of fiscal policy in HANK economies, the structure of the asset market can be crucial.

7.2 Inspecting the mechanism

Above, we isolated a succinct "debt inflation" effect by leveraging the asset market structure presented in 4.4. What are the economic forces at work? As suggested previously, a particularly simple way to rationalize the occurrence of the "debt inflation" is by considering the Taylor rule jointly with the Fisher equation. Yet, it may also feel somewhat unsatisfactory, as this explanation is hard to fit into typical narratives of inflation arising from there being too much demand for too little supply (or vice versa). From a general equilibrium perspective, higher real returns being required for asset market clearing also means that without, households would put more money into consumption or illiquid capital goods, raising demand for actual output. By analyzing how the response of the aggregate household sector is determined, we thus get an idea where "excess demand" would arise if the central bank would not engineer higher real liquid returns in response to inflation.

For this purpose, I use the Sequence Space Jacobians of the 2-asset model's household block to decompose how much of the household sector's first-order response to the shock is due to the various prices and policies the household is *directly* affected by. For example, the contribution of transfers to consumption indicates how much household consumption would have changed if it only received the transfer but all interest rates, wages, etc., remained at the steady state level.

The results for the Baseline economy are displayed in Figure 9: Panel 9a re-affirms that the aggregate consumption response to the transfer shock is largely independent of interest rate dynamics and instead mostly determined by the initially and subsequent declines of real post-tax wage incomes. While the latter depend on inflation through general equilibrium effects, a comparison with the corresponding figure for the $\Psi = 0$ economy (Figure E.5 in Appendix E) suggests such effects to be small. While the responses for liquid savings in Panel 9b do not correspond to demand for the economy's output, we can confirm the importance of higher liquid returns to achieve asset market clearing after the shock. Additionally, 9c indicates that these are also important through their effects on household portfolio choice, i.e., the higher demand for investment goods is not just driven by the LAF's portfolio choice but also by households directly: If liquid savings provides higher returns, it becomes relatively more attractive to hold more for self-insurance purposes.

As a take-away, the above decomposition corroborates capital demand to be a key margin for the additional inflationary pressure in the baseline economy, as the aggregate responses already alluded. This also means that while simple incomplete markets models point to related effects, the quantitative analysis of fiscally-driven natural rates and their inflationary consequences appears to require more sophisticated models featuring illiquid investment goods and household portfolio choice.

8 Robustness Analysis

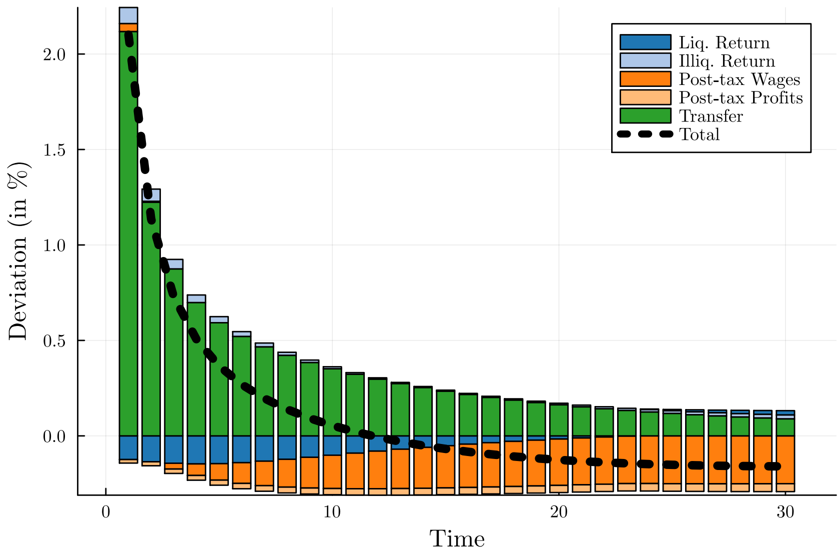
The previous Section 7 established that in quantitative HANK frameworks, public debt can exert quantitatively relevant pressure on inflation through its effect on equilibrium interest rates and investment demand. Yet, the respective model exercises were conducted only in the context of a particular fiscal policy scenario and the model features several parameters whose values are subject to empirical controversies. Therefore, before moving on to a more complex application, this Section aims to address concerns regarding the generality and robustness of the previous results.

8.1 Alternative Fiscal Policy

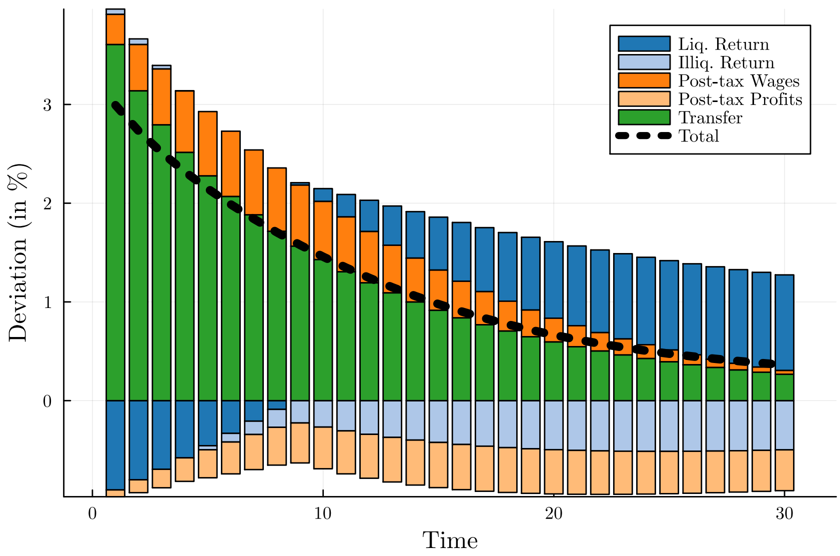
Firstly, to what extent does the previous section's result depend on the used specification of fiscal policy? To answer this question, I re-solve the model under various combinations of the parameters ρ_τ and ψ_B that determine the persistence of the tax level and its responsiveness to the value of public debt, respectively.[24] For the same transfer shock as in the previous Section 7, Figure 10 indicates for each of these combinations the induced average deviation of public debt during the first 40 quarters (10 years) as well as cumulative inflation under the that period: The results for the Baseline HANK model are displayed in the upper panels and the ones for the $\Psi = 0$ counterfactual in the lower ones. Clearly, combinations with high ρ_τ and low responsiveness ψ_B result in higher public debt. In the

[24] All considered combination result in stable and determinate equilibria.

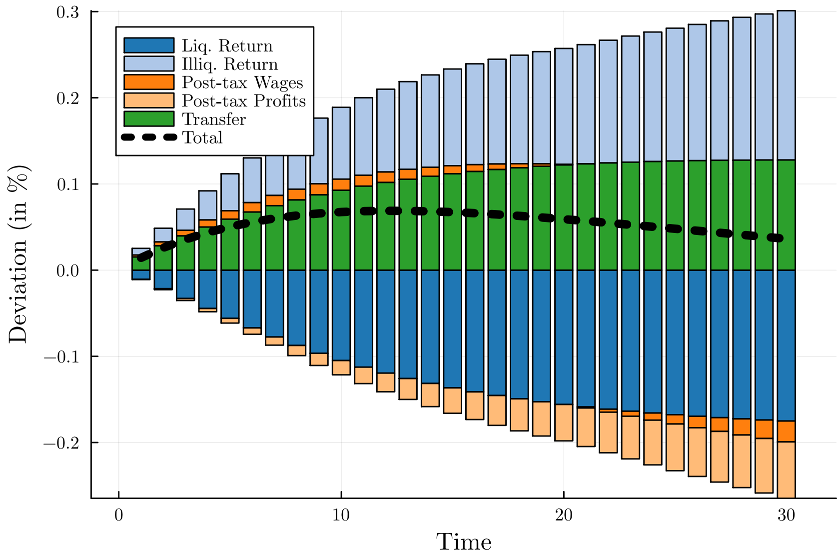
Figure 9: Decomposition of household responses to Transfer Shock



(a) Consumption Response

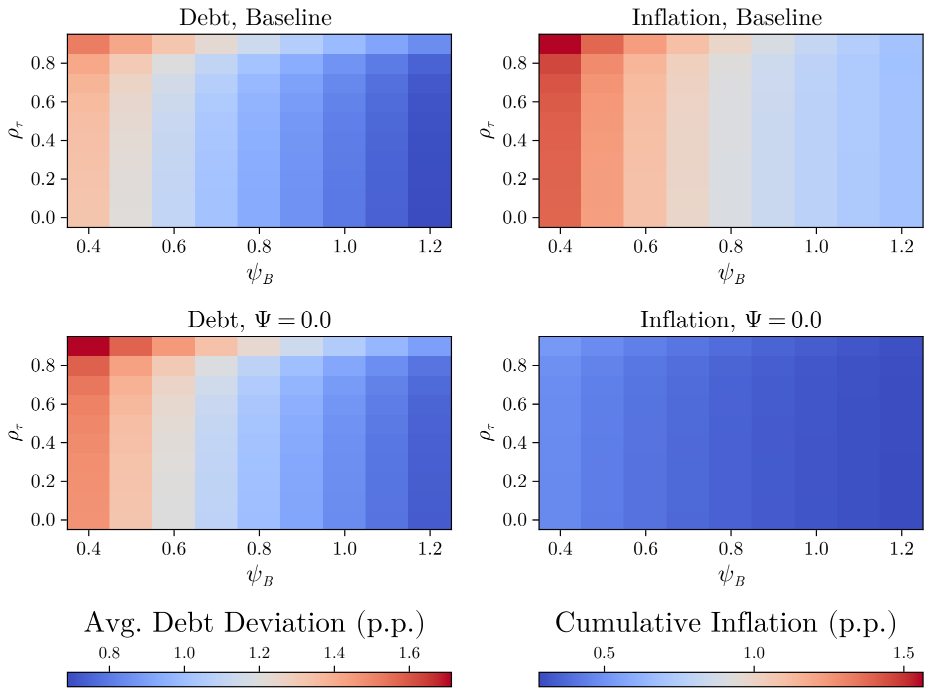


(b) Liquid Savings Response



(c) Illiquid Savings Response

Figure 10: Debt and Inflation under alternative fiscal policy



Note: The panels on the left-hand size indicate the average percentage-point (p.p.) deviation of public debt (market value) relative to annualized GDP during the first 40 quarters after a one-time transfer shock as in Section 7. The panels on the right-hand side indicate cumulative inflation during the same period.

Baseline economy, this is systematically associated with noticeably higher increases in the price level. In comparison, in the $\Psi = 0$ case with very small effects of public debt supply on liquid interest rates, we always see lower inflation and a much less pronounced association with public debt. We can thus reaffirm that under conventional Taylor rule-type monetary policy, the time path of public debt constitutes an important determinant of inflation and that this depends on its potential effects on real liquid asset returns.

Additionally, I also considered the inflationary effects of public debt under different fiscal shocks or under alternative consolidation policy: The resulting figures are delegated to Appendix E for the sake of brevity. In particular, Figure E.3 displays the macroeconomic response to a persistent increase in government consumption G, assumed to increase by 2.5% on impact and afterwards following an AR(1)-decay with persistence 0.9. Figure E.4, in turn, presents the aggregate response to the transfer shock if the fiscal authority subsequently reduces debt by lowering government consumption G_t and leaves taxes unchanged. The overall responses for the former case align with previous work, featuring approximately a unit multiplier on impact and eventually declining consumption due the shock's wealth effect and negative impact on investment. In either case, we again notice inflation and the policy rate to remain persistently elevated compared to the $\Psi = 0$ case with integrated asset markets, accompanied by less investment decline. Hence, it appears that our previous insights also apply to other fiscal policy shocks or alternative means of public debt consolidation.

8.2 Alternative Parameterizations

Like every New Keynesian model, the framework used for the above analysis contains several parameters subject to empirical controversies, in particularly the "slopes" of the New Keynesian Price- and Wage Phillips curves. Additionally, since investment demand was found to be important for the transmission of the debt-driven inflation, it is relevant to check to whether the above results are due to specific values for the model's capital utilization- and adjustment costs. To address this, I varied the parameters one-by-one to assert robustness with respect to the Baseline calibration's parameter choices. The resulting IRFs to the transfer shock are visualized in Appendix E and briefly summarized below.

Allowing for higher or lower price stickiness by varying the NKPC slope κ affects the absolute size of the initial inflation response to the fiscal shock, but not so much the eventual amount of natural-rate driven inflation persistence. The same is the case for wage stickiness, even though we notice even bigger differences in the real response to the shock (c.f. Figures E.6 and E.7). Results are similarly robust to considering alternatives values for the investment adjustment- and utilization parameters, alternative values for which give rise to somewhat different time paths of inflation but only small differences in "debt inflation" (Figures E.8 and E.9).

9 Debt-driven inflation in the US post-2020?

Above, we found public-debt driven upward pressure on the neutral rate to be a noticeable driver of inflation in the aftermath of fiscal shocks. While its moderate magnitude may perhaps be taken as not being an important consideration in normal times with restrained policy, this section demonstrates that it is in the aftermath of volatile episodes featuring strong and sudden fiscal expansions. In particular, it investigates the potential importance of public debt-driven inflation pressure in the context of the USA during the post-2020 period.

9.1 Methodology

To analyze and conduct counterfactuals for this episode, we need the model to generate a situation in line with the economic dynamics of that time. To do so, I use the filtering algorithm proposed by McKay and Wieland (2021) to obtain sequences of the model's 5 business cycle shocks that make the framework match the evolution of 5 aggregate variables during the period 2020:Q1-2024:Q2. The method can account for an ELB and is described in more detail in Appendix B.3. Recall that the HANK model's 5 shocks contain two supply shocks (investment technology, "cost push"/markup), a demand shock (discount factor) and two policy shocks (monetary policy, transfers). Under the assumption that

	Z^I	μ	ϵ^R	A	T
AR(1) persistence	0.75	0.9	0.0	0.9	0.5

Table 5: Assumed persistence of model shocks

the economy was in steady state in 2019, these exogenous disturbances will be used to replicate the subsequent evolution of aggregate output, investment, inflation, the central bank's policy rate as well as government transfer spending. For the real variables subject to trend growth, this refers to the relative deviation from their pre-pandemic trends instead of levels.[25] For further information on the construction of the targeted variables, please refer to Appendix C.

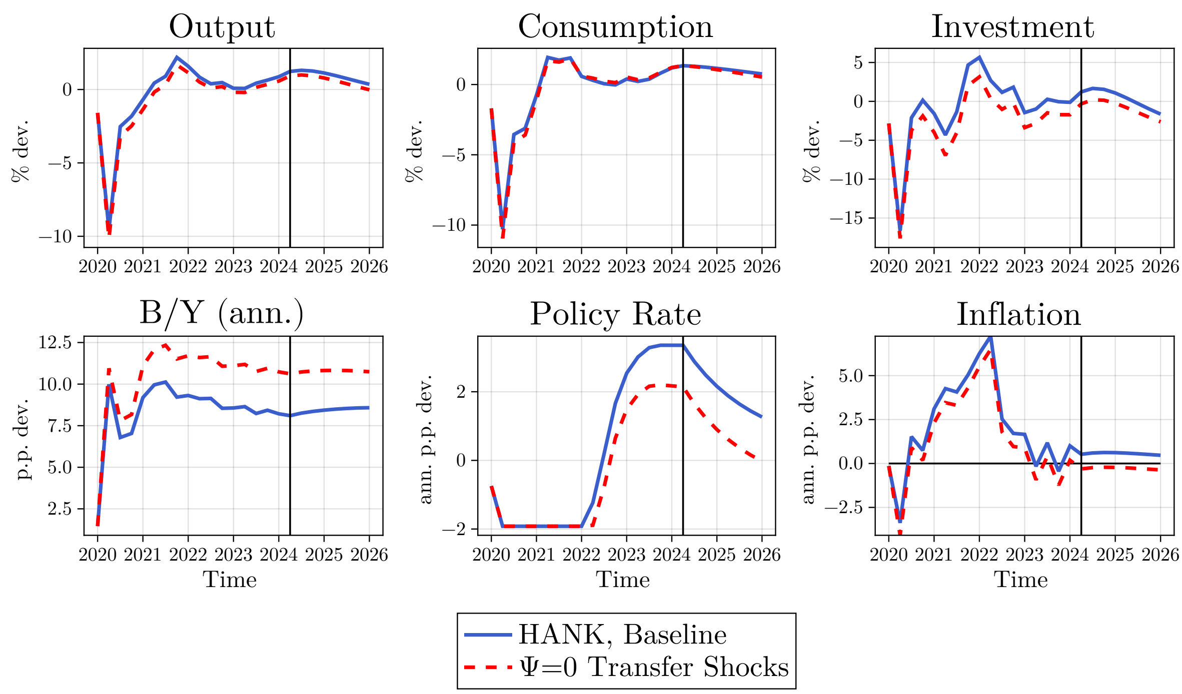
The choice of the 5 business cycle shocks mentioned above is motivated as follows: The discount factor shock is supposed to induce pandemic-related consumption restraints as in Bardóczy et al. (2024), which requires additional investment shocks to not give rise to counterfactual higher capital accumulation. The monetary policy- and transfer shocks are needed to replicate the time-paths of the policy rate and transfer payments while the "cost-push" shock captures the remaining variation in inflation. I intentionally do not model other Covid-related spending programs such as support for corporations as it is less clear to whom these should be assigned in my framework. Given that this will result in my model generating lower public debt levels than in the data, I view this as a conservative choice.

Since the number of shocks equals the number of target variables, the McKay and Wieland (2021) filtering method does not require me to take a stance on the variance of the business cycle shocks. However, assuming that all shocks follow $AR(1)$-processes, I still need to make assumptions on their persistence in order to compute the necessary IRFs. My calibration choice is presented in Table 5: The values for the supply shocks (μ, Z^I) and the discount factor disturbance (A) are set to salient values in the ballpark of Bayer et al. (2024)'s estimates. Since many of the big fiscal expansions during the pandemic period were designed to be short-lived, I further assume the autoregressive parameter of the transfer shock to be a low 0.5. As the rate smoothing term of the monetary policy rule (33) already provides for a persistent impact of the rate shock ϵ^R, the latter does not depend on its previous value.

Naturally, the resulting analyses will have some caveats worth discussing. First of all, my model doesn't feature any Covid-related features such as lock-downs but rather assigns the observed aggregate dynamics during 2020-2021 to standard business cycle shocks. While in line with other DSGE model-based investigations of the post-pandemic inflation (e.g. Gagliardone and Gertler, 2023; Bianchi et al., 2023), the model does thus only provide a simplistic account of various pandemic-specific phenomena. This includes the nature

[25] Strictly speaking, I cannot match transfer spending's relative deviation from trend in my model as transfer payments are zero in its stationary equilibrium. Instead, I match the deviation of transfers relative to trend output.

Figure 11: Aggregate dynamics using filtered shocks



Note: B/Y represents the market value of public debt B^g over annualized GDP.

of transfer payments made by the government, which are assumed to consist solely of uniform lump-sum payments for the purpose of this exercise: Since transfers specifically aimed at poor agents with high MPCs tend to have larger effects in HANK models, this simplification can again be seen as a conservative choice. Secondly, the fact that the analysis is based on a linearized model means that we will miss out on non-linearities that may be relevant for the large shocks occurring during the period under consideration. Again, my analysis shares this reservation with numerous other studies (including the two cited previously). Finally, all the results obviously depend on the assumption of specific policy rules: Under different ones, e.g., a partly active fiscal policy regime as in Bianchi et al. (2023) instead of the active Taylor rule, the same aggregate dynamics might be assigned to different shocks.

9.2 Results: Model dynamics

Using the set of aggregate shocks obtained as described above, we can now simulate the model from 2020:Q1 forward: The dynamics of several key macroeconomics aggregates are displayed as the blue solid line ("Baseline") in Figure 11. Note that by construction, the time paths of all displayed variables except the public debt variable (B/Y) and consumption equal their counterparts in the data until my sample ends in 2024:Q2 (indicated by the black vertical line). Beyond that point, the model is simulated forward without any additional shocks hitting.

In the beginning of 2020, we see real variables such as Output and Investment take a big hit, accompanied by declining inflation and the Central Bank's policy rate hitting its ELB. Through the lens of the model, this is mostly due to a combination of households' Covid-related consumption restraints (i.e. the discount factor shock) and an unfavorable investment technology disturbance. At the same time, public debt relative to GDP jumps up, both due to the decline in the denominator and a big shock to transfers, the targeted time path of which is displayed in the top-right panel. Of course, as I will illustrate in Section 9.4 below, the model increase in debt is smaller than its relevant counterpart in the data, given that I only target transfer spending.

Afterwards, the economy recovers quite quickly: In the HANK model, this is partly due to the initial shocks easing quicker than expected, i.e., expansionary discount factor- and investment technology innovations, and also aided by accommodative monetary policy as well as another spike of transfers at the beginning of 2021 (the American Rescue Plan Act). Eventually, inflation starts to rise precipitously amid output and consumption above their pre-covid trends, inducing the Federal Reserve (Fed) to start raising its nominal rate in the beginning of 2022. Afterwards, price pressures ease quickly initially, but inflation remains elevated above target and is indeed predicted to do so for quite some time into the future. At the same time, the model's value of public debt relative to GDP has not substantially declined since its peak at the beginning of the pandemic.[26] Naturally, by now we expect the final two phenomena to be linked.

9.3 Determinants of Inflation

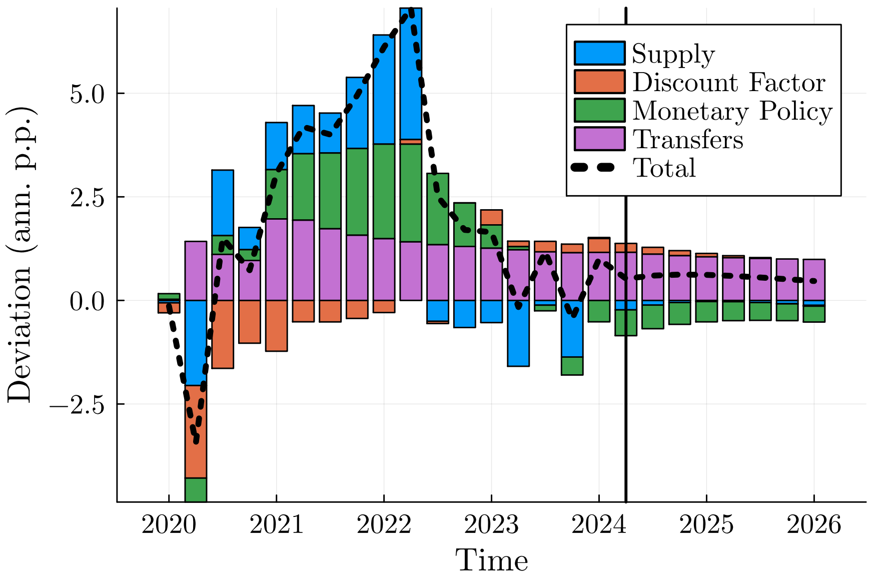
To get a better understanding to what extent the neutral-rate driven "debt inflation" is related to this observation, Figure 12 exploits the linearity of my model solution to decompose the model's inflation response into the contribution of its different shocks. As already explained above, the deflation at the beginning of the Covid pandemic is due to a combination of discount factor- and supply shocks, partly counteracted by the strong increase in transfers.[27] Interestingly, while the McKay and Wieland (2021) filtering method assigns the 2022 peak in inflation to adverse supply shocks, it also suggests the combination of government transfers and accommodative monetary policy to be the key drivers of inflation during 2021-2022. My model exercise thus supports the findings of Giannone and Primiceri (2024) who argue such "demand side" factors to be the most important determinants of the post-Covid price pressures. Incidentally, the strong decline of inflation in 2022:Q3 is interpreted to be due to an unexpected easing (negative innovations) of the "cost push"-shocks, perhaps reflecting the decrease of the oil price and an easing of supply chain bottlenecks at the time.

From the perspective of this paper's topic though, the most interesting observation is the

[26]Under the fiscal rule in place, public debt starts declining in late 2026. For a graph with a longer simulation horizon, see Appendix Figure E.11.

[27]The negative contribution of Monetary Policy reflects the binding nominal interest ELB.

Figure 12: Decomposition of inflation



Note: "Supply" collects the impact of both the "cost-push"- and the investment technology shock.

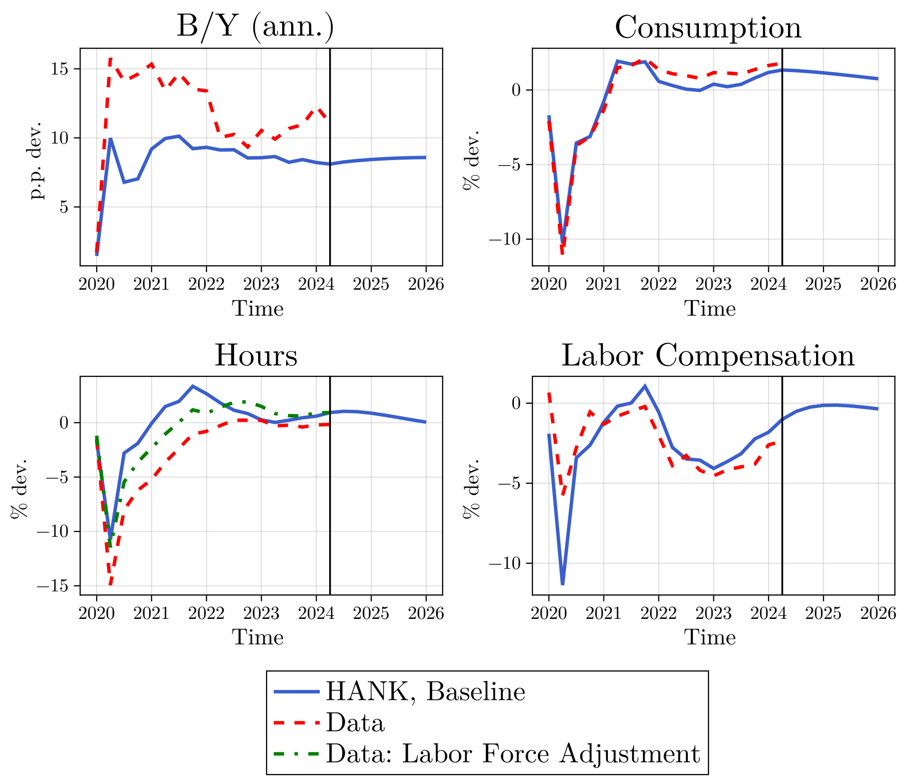
persistent impact of the transfer shocks. Additionally, considering Figure **??**, we see that they retain a strong influence on inflation for an extended period of time after their peaks in 2020 and 2021. Indeed, the model suggests them to be the sole reason for inflation staying above target after 2023 and the continuing upward pressure on inflation going forward. To what extent is this due to transfers remaining above trend at the time as compared to the interest rate pressure exerted by the public debt stemming from previous transfers?

To gauge the importance of the transfer-induced debt as compared to current transfer levels, I again make use of the linearity of my model solution, which provides for a $MA(\infty)$-representation in that the time path of a variable model variable x_i can be expressed as

$$x_{it} = \bar{x} + \sum_{e=1}^{n_e} \sum_{i=0}^{\infty} \Theta_x^e(i) \epsilon_{t-i}^e \quad ,$$

where $\Theta_x^e(i)$ denotes the i'th entry of the IRF of variable x with respect to shock e. Specifically, I simulate my model using that representation but propagate the transfer shocks according to the "wrong" IRF from the $\Psi = 0$ economy with integrated asset markets: As for the simple scenario in Section 7, this attempts to (almost) "shut down" the effect of transfer-related public debt supply on liquid asset returns. The result of this exercise as displayed as the red-dashed line in Figure 11 and we see that in the absence of that channel, the post-Covid inflation would have practically been over in the beginning of 2023 and remain at or below the Fed's target afterwards. This happens despite a less pronounced central bank reaction, as less inflation is needed to increase. Again as in the simple scenario, the $\Psi = 0$ case is also associated with lower investment and more pronounced increase in the value of government debt.

Figure 13: Non-targeted variables: Model vs. Data



Note: B/Y represents the market value of public debt B^g over annualized GDP.

We can further back up that conclusion by re-doing the entire exercise for the $\Psi = 0$ economy: In that case, the corresponding inflation decomposition (relegated to Appendix Figure E.12) assigns much less importance to the transfers but rather attributes the realized inflation to other shocks. It also predicts less inflation persistence going forward. Overall, we can thus conclude that the public debt-driven interest rate pressure has the potential to exert relevant inflationary pressure in the aftermath of big fiscal expansions, to the extent that it could quantitatively explain most of the US's post-2023 inflation persistence.

9.4 Comparison for untargeted variables

Before moving on, it is useful to assess how well the model relates to some non-targeted moments so as to be able to judge what aspects of the economy it does or does not capture well. Again, the construction of the additional data used here is specified in Appendix C. The top-left panel of Figure 13 compares the relative value of public debt in the model with an approximation of the market value of domestically held US federal debt in the data. As anticipated, the model generates a smaller expansion, implying that if anything, my exercise will have under-estimated the amount of "debt inflation".

While aggregate consumption was not directly targeted in the construction of the shocks, the resulting fits the data very well: This is not surprising as in the model (and reality), the by far most important components of GDP are private investment and consumption. Having targeted both Output and Investment, a good fit for Consumption is essentially by construction.[28]

An relevant and more interesting set of non-targeted model variables relates to labor supply. Here, the success of my model partly depends on which data moment one would consider the most relevant real-world counter-part for the simple set-up in my HANK model: In the center panel of Figure 13, we observe that if one follows the common convention of using Hours Worked divided by the aggregate population (red-dashed line), my model substantially over-estimates the labor supply recovery after the pandemic. In contrast, if adjusting by the size of the civilian labor force (green dot-dashed line), it does a better job.

Given that the conventional DSGE labor market set-up in my model is arguably too simplistic to capture details such as time-varying participation and composition-effects important during the pandemic recovery, a perhaps more reasonable demand is it matching well the overall amount of labor compensation paid ($h_t H_t$ in the model), which directly matters for the model households' aggregate consumption- and savings decisions. While my model overstates the initial drop at the beginning of the pandemic, it matches its subsequent dynamics well, exuding confidence that the HANK framework captures the relevant economic forces at work at least after 2020.[29]

Finally, one may be wondering whether the model's forecast of elevated inflation going forward is at odds with later developments after 2024:Q2. On this issue, recall that the forward simulation does not account for subsequent shocks hitting the economy and any further disturbances could of course explain the difference. Indeed, from the perspective of 2024:Q2, inflation expectations in my model do not seem unreasonably out of touch with various measures of inflation expectations at the time: According to the Cleveland Fed's model of inflation expectations, 1- and 2-year expected inflation was approx. 2.7% and 2.6% in June 2024, respectively. The May 2024 Survey of Professional Forecasters suggests an expected inflation of 3.1% for 2024 and 2.5% for the period 2024 - 2028, while FOMC member's inflation expectations ranged between $2.5 - 3.0\%$ for 2024 and $2.2 - 2.5\%$ for 2025 at the time (c.f. Federal Open Market Committee, 2024). For comparison, my model's predicted inflation rate is approx. 2.7% for 2024 and 2.6% for 2025.

[28]The fact that the match is not perfect is due to the variables being de-trended separately and the dynamics of government consumption not being targeted.

[29]The small initial drop in labor compensation despite the strong fall in hours at the beginning of the Covid-pandemic is due to composition effects, with much more low-income than high-income workers being laid off.

10 Implications for Monetary Policy

After the analyses relating public debt supply to inflation under Taylor rule-type monetary policy, the question in which ways such feedback should and could be prevented obviously arises. In this Section, I shall briefly elaborate on the latter, noting that welfare-optimal policy may involve complex distributional considerations in HANK models such as this. If preventing the "debt inflation" is supposed to be achieved by building on a parsimonious policy rule, the following options promise to specifically address the issue of a debt-driven time-varying natural rate:[30]

1. **Public debt reaction:** Recall that the cause of the liquidity-supply driven inflation is that, under a Taylor rule, higher real interest rates on liquid assets can only come about if inflation is also higher. This mechanism is broken if, in response to higher public debt, the central bank sets a higher nominal rate at *any* given inflation level, i.e. it directly reacts to also raise the value of public debt.
For the purpose of this section, I shall implement this idea by replacing the $\log(1 + r_{ss}^R)$-terms in the HANK model's interest rule (33) with the approximated "debt-corrected" natural rate as in (39). While straightforward in theory, employing such a rule in practice requires knowledge of the elasticity of the "natural" rate with respect to government debt, estimates of which are of course available but associated with some uncertainty. Note that such a debt-adjusted rule would require the central bank to raise the nominal rate even if higher public debt is *not* accompanied by any inflation.

2. **Difference rule:** Another possibility might be the so-called Difference rule originally proposed Orphanides and Williams (2002) and suggested by Campos et al. (2024) to address public debt's interest rate effects. These authors formulate it as

$$\log(1 + r_{t+1}^R) = \log(1 + r_t^R) + \theta_\pi \left(\log(\pi_t) - \log(\pi_{SS})\right) \quad , \tag{40}$$

i.e. it resembles a Taylor rule with the previous nominal rate replacing the usual long-term r^*: This has the appealing property of not requiring any knowledge of a neutral rate whatsoever. It is worth noting, though, that despite the similar appearance, it implies a quite different conduct of monetary policy. In particular, the rule requires the central bank to never cut the nominal rate as long as inflation remains above target (but keep raising it instead).

For the quantitative evaluation, I follow Campos et al. (2024) by parameterizing the Difference rule in the same way as the Taylor rule, i.e. $\theta_\pi = 1.5$.

[30] In standard New Keynesian models, an active monetary authority can always bluntly suppress any type of inflation by adopting a strict inflation-targeting regime or the type of rules studied by Holden (2024).

For the sake of brevity, the response to the transfer shock as in Section 7 under the alternative rules is relegated to Figure E.13 in the Appendix. It seems that either rule effectively counteracts the inflationary pressure of public debt: Specifically, the Taylor rule adjusted with a public debt reaction (red-dashed line) implements an outcome quite similar to the $\Psi = 0$ economy, featuring less inflation for a lower policy rate. On the other hand, the difference reduces the shock's inflationary impact even more at the cost of a bigger drop in investment and a somewhat reduced stimulus on impact. In this simple scenario, either rule yields seemingly reasonable results, the desirability of which of course depends on policymakers' preferences on trading off inflation and real activity.

10.1 The importance of explicit rules

An interesting question is to what extent central banks may already be counteracting public debt's effect on the natural rate by following a policy akin to the debt-adjusted Taylor rule.[31] After all, it seems desirable in achieving lower inflation at lower nominal rates and central bank policymakers do not explicitly commit to certain rules that also happen to be notoriously difficult to identify empirically.

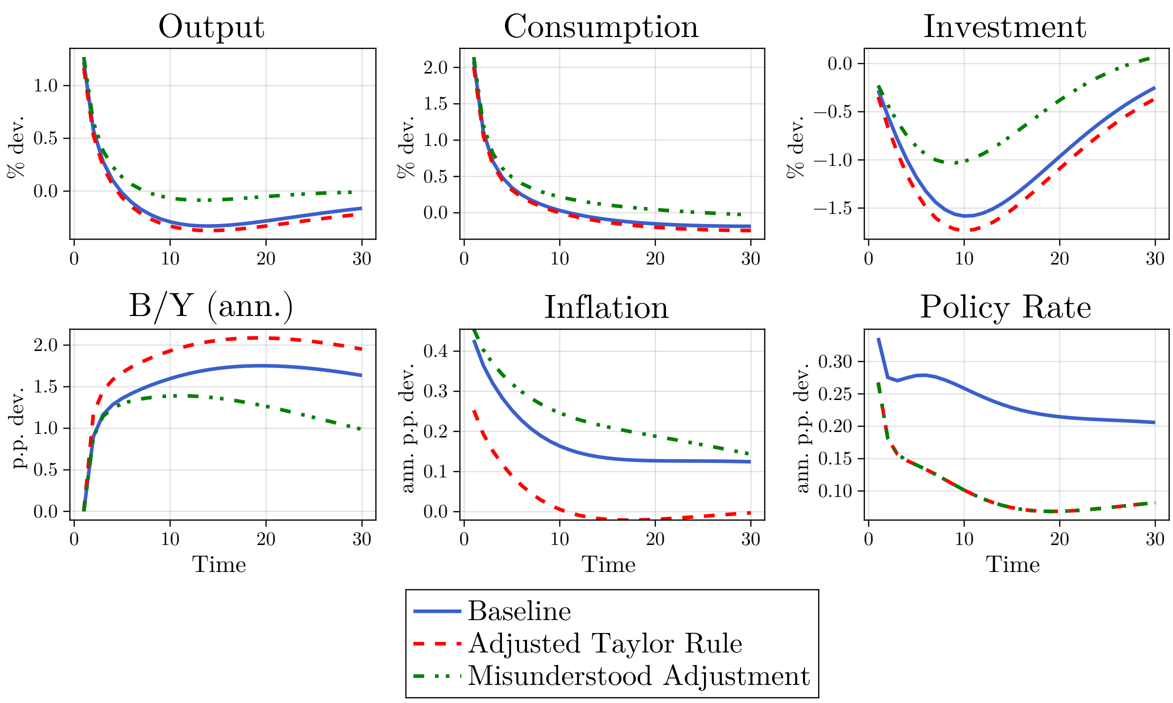
However, this being unclear indicates that the debt adjustment may not work as well if it is actually intended. To make this case formally, I consider what would happen if the central bank chose to implement the nominal rate path consistent with the debt-adjusted Taylor rule after the transfer shock but agents do not understand this and expect the monetary authority to follow the Baseline Taylor rule instead: Formally, this is equivalent to the Baseline rule being in place and a series of monetary policy surprises (from the perspective of the private sector) causing the nominal rate to align with the one from the red-dashed response for the *understood* adjusted rule. The results of this experiment is displayed as the green dash-dotted line in Figure 14: If the path consistent with the successful debt adjustment is implemented in an economy in which agents misunderstand it to be a deviation from a different rule, the inflation persistence becomes even more pronounced while the perceived monetary policy surprises provide additional stimulus to the economy. While this example is perhaps extreme in that the entire private sector is assumed to be ignorant about the true rule, it indicates that achieving their intended effects requires policies such as the "debt adjustment" to be openly communicated.

10.2 Alternative policy rules post-2020

Given the results of 9, it is finally interesting whether the alternative policy rules might have prevented the post-2022 inflation compared to the baseline model. To assess the performance of the alternative rules, I consider the following scenario: In the Baseline

[31] Given the discussed properties of the Difference rule, it is clear that most major Central Banks do not operate according to its predictions.

Figure 14: Response to transfer shocks: Misunderstood Adjustment



Note: B/Y represents the real market value of public debt B^g over annualized GDP.

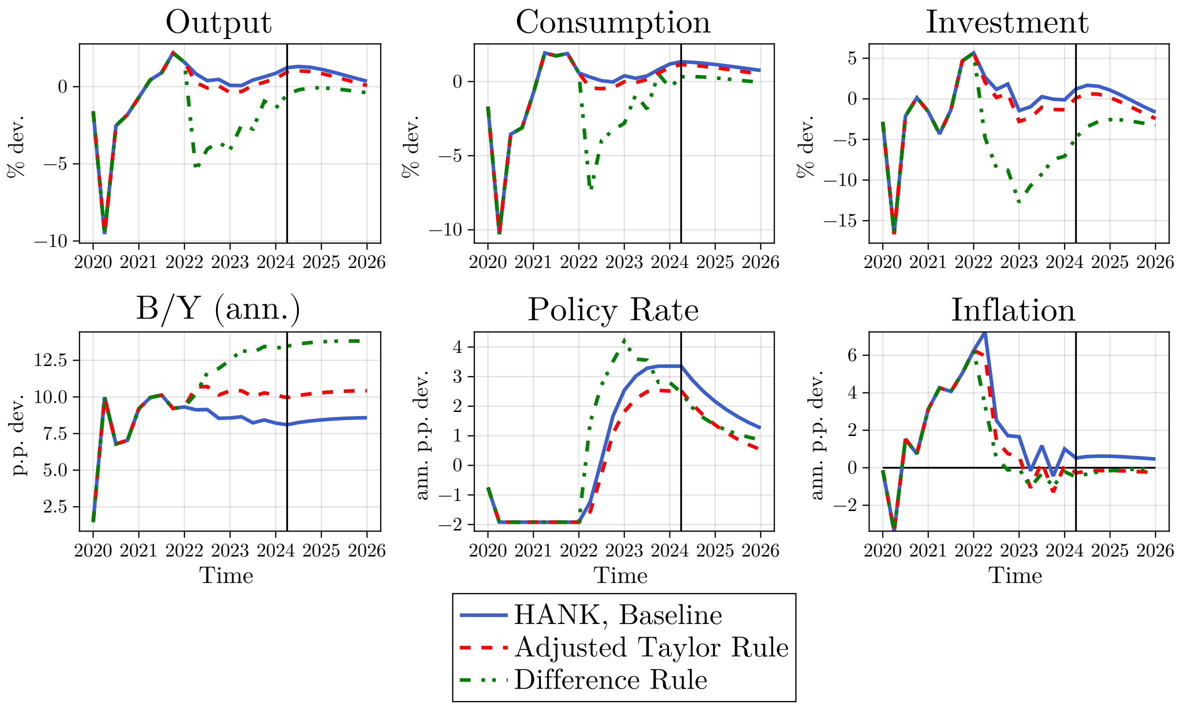
HANK subject to the shocks obtained as in Section 9.1, the interest rate rule switches to one of the alternative rules as of 2022:Q1 when interest rates started to rise after the pandemic, perhaps reflecting policymakers' concerns about fiscal influence on the neutral rate going forward. The chosen time allows me to sidestep the issues that a) the policy shocks were not uniquely identified due to the ELB before and b) the fact that my model might not capture the initial pandemic recession in 2020 that well.[32]

How well would that have counteracted the suggested "debt inflation" and at what costs? The results as displayed in Figure 15 shows that in the model, implementing the Taylor rule adjusted for the natural rate effects of public debt supply (red-dashed line) results in a outcome very similar to the counterfactual conducted in Section 9.3. In particular, it allows the central bank to avoid elevated inflation post-2023 while raising the nominal rate less. Naturally, this outcome assumes the private sector to immediately understand the rule change and entails limited costs in terms of output and an actually higher value of public debt. The latter is again due less pronounced devaluations.

In contrast, while the difference rule is able to eliminate inflation rapidly, it induces stark declines of consumption and investment amid a rapidly rising nominal rate. While I do not evaluate welfare and central banks are typically assumed to be willing to accept some costs in real activity to counteract rising price levels, it seems questionable that policymakers

[32]Sticking to their interpretation as structural shocks, I assume the economy to still be subject to the same monetary policy shocks after the switch. However, my conclusions for the Difference rule with $\theta_\pi = 1.5$ wouldn't change much otherwise.

Figure 15: Post-2020 aggregate dynamics under alternative rules



Note: B/Y represents the market value of public debt B^g over annualized GDP.

would have preferred this outcome to the Baseline. What explains this results? Actually, the difference rule is very "hawkish" if calibrated with the same θ_π as a Taylor rule: Reformulating it as

$$\log(1+r^R_{t+1}) = \rho_R \log(1+r^R_t) + (1-\rho_R)\left(\log(1+r^R_t) + \frac{\theta_\pi}{1-\rho_R}\left(\log(\pi_t) - \log(\pi_{SS})\right)\right)$$

to resemble a Taylor rule with nominal rate persistence indicates that the latter's corresponding inflation reaction would be a very strict $\frac{\theta_\pi}{1-\rho_R} = 7.5$ under a standard $\theta_\pi = 1.5$ and $\rho_R = 0.8$ parameterization. Since the Difference rule does not require the familiar condition $\theta_\pi > 1$ to ensure determinacy, one can alternatively consider the parameterization $\theta_\pi = (1-\rho_R) \times 1.5 = 0.3$. This yields mixed results: If subjected to the same monetary disturbances as the Baseline interest rule, it would induce very persistent but less severe output losses, but if not, result in an outcome somewhat similar to the debt-adjusted Taylor rule (see Appendix Figures E.14 and E.15, respectively). This likely reflects that a Difference rule reacting little to the current situation but necessarily depending strongly on the previous policy stance will have to "carry around" monetary policy shocks for a long time and thus exacerbate their effect.

From the above exercises, it seems that while the Difference rule may prove beneficial in simple New Keynesian models or in the face of specific demand side shocks, it does less obviously so in richer frameworks subject to more frictions as well as inefficient "cost-push" shocks. While it holds some conceptual appeal, my results indicate that its properties and proper calibration require at best further investigation before it is ready to provide guidelines for tackling the issue of a fiscal policy-dependent natural rate. Making an established operating procedure sufficiently responsive to government debt supply and ensuring that this is widely understood may be a more practical option, although its welfare effects should similarly be investigated.

11 Concluding Remarks

This paper revisited the question whether high public debt levels matter for monetary policy and found that it does, even if fiscal policy is committed to eventual budget consolidation: A tractable pen-and-paper exercise revealed that if government debt obligations are useful for private sector agents to insurance against idiosyncratic risk, public debt supply will in general affect "neutral" interest rates and cause inflation if central banks operate according to standard Taylor rule-like monetary policy. Motivated by relevant evidence, I analyze the issue quantitatively using a 2-asset HANK framework disciplined to provide for long-run effects of public debt on interest rates in line with various empirical estimates. The Baseline model suggests that public debt may induce moderate but potently quite persistent upward pressure on inflation and is thus most relevant in the aftermath of big fiscal expansions: Indeed, I find that in the case of the US, the aggregate demand effects stemming from the level of public debt itself can potentially explain elevated "last mile" inflation in 2023 and afterwards. Given that fiscal policy is likely to respond forcefully also to future crises, it seems thus important for central banks to consider how such effects could and should be addressed. While I briefly evaluate some ideas on the former, the latter remains an open question. Additionally, my paper highlighted asset market arrangements to be a crucial determinant of nominal and real dynamics in rich HANK frameworks. While I address this with a simple model extension, micro-founding their asset market structure more rigorously may provide for interesting and novel policy implications in future work.

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A Proofs for analytical model

A.1 Proof of Proposition 1

Let us first consider the First order condition for labor supply of an household of type $i \in \{h, l\}$:

$$\frac{w_t z^i}{c_t^i} = \gamma \frac{1}{1 - N_t^i} \Leftrightarrow (1 - N_t^i) w_t z^i = \gamma \left(w_t z^i N_t^i + \frac{(1 + i_t)}{\pi_t} b_{it-1} - b_{it} - z^i \tau \right) \tag{41}$$

$$\Leftrightarrow N_t^i = \frac{1}{1+\gamma} \frac{1}{w_t z^i} \left(w_t z^i + \gamma (b_{it} + z^i \tau - \frac{(1 + i_t)}{\pi_t} b_{it-1}) \right) \tag{42}$$

where the c_t is substituted using the budget constraint in the second step. Summing up the labor supplied by both groups yields aggregate labor supply

$$\begin{aligned} N_t &= \rho^h z^h N_{ht} + (1 - \rho^h) z^l N_{lt} \\ &= \frac{1}{1+\gamma} \frac{\rho^h}{w_t} \left(w_t z^h + \gamma (b_{ht} + z^h \tau - \frac{(1 + i_t)}{\pi_t} b_{ht-1}) \right) \\ &+ \frac{1}{1+\gamma} \frac{1 - \rho^h}{w_t} \left(w_t z^l + \gamma (b_{lt} + z^l \tau - \frac{(1 + i_t)}{\pi_t} b_{lt-1}) \right) \\ &= \frac{1}{1+\gamma} \end{aligned} \tag{43}$$

The second step follows uses (1), bond market clearing condition

$$b_t^g = 0 = \rho^h b_{ht} + (1 - \rho^h) b_{lt} \quad \forall t \geq 1$$

and in turn also $\tau_t = \frac{1+i_t}{\pi_t} b_{t-1}^g$ by the government budget constraint (7) and the assumed policy path.

Next, it is easy to see that either group of household's intertemporal Euler equation will be of the form

$$c_{it+1} = \beta \frac{(1 + i_{t+1})}{\pi_{t+1}} c_{it} \quad . \tag{44}$$

Using again the budget constraints, we further obtain

$$\begin{aligned} &\rho^h c_{ht} + (1 - \rho^h) c_{lt} = \\ &\rho^h \left(w_t z_h N_{ht} + \frac{(1 + i_t)}{\pi_t} b_{ht-1} - b_{ht} - z_h \tau_t \right) \\ &+ (1 - \rho^h) \left(w_t z_h N_{ht} + \frac{(1 + i_t)}{\pi_t} b_{ht-1} - b_{ht} - z_h \tau_t \right) \\ &= \frac{w_t}{1+\gamma} \end{aligned}$$

after again using (1), (7), the bond market clearing condition and additionally (43). Naturally, absent profit incomes and capital investments, total household consumption must equal household labor income.

In turn, we can sum up (44) over types to obtain

$$\begin{aligned} \rho^h c_{ht+1} + (1-\rho^h) c_{lt+1} &= \beta \frac{(1+i_{t+1})}{\pi_{t+1}} (\rho^h c_{ht} + (1-\rho^h) c_{lt}) \\ \implies w_{t+1} &= \beta \frac{(1+i_{t+1})}{\pi_{t+1}} w_t \end{aligned} \tag{45}$$

Together with (6) and (5), (45) forms a system that characterizes the equilibrium of the model for $t \geq 1$. Since $r_t^* = \frac{1}{\beta} - 1 \quad \forall t \geq 1$, it is easy to verify that $\pi_t = 1$, $w_t = \frac{\epsilon - 1}{\epsilon}$ and $i_t = r_t^*$ indeed solve that system. Local stability of this equilibrium is ensured by the Taylor principle.

Also, from the Euler equation it follows that in such an equilibrium, the consumption either household type $i \in \{h, l\}$ will be constant over time. Hence, we can back out their savings choice from the budget constraint for $t = 1$ and $t = 2$:

$$\begin{aligned} c_1^i &= c_2^i \\ \Leftrightarrow w_1 z^i N_{ss}^i + (1+i_1) b_{i0} - b_{i1} - z^i \tau_1 &= w_2 z^i N_{ss}^i + i_{ss} b_{i1} \\ \Leftrightarrow b_{i1} &= \frac{1}{1+i_{ss}} \left((1+i_1) b_{i0} - z^i \tau_1 \right) \end{aligned}$$

The second step uses that for $t \geq 1$, we need to have $b_{it+1} = b_{i+2}$ and since $c_{it} = c_{ss}$ as well as $w_t = w_{ss} \forall t \geq 1$, also $N_t^i = N_{ss}^i$ due to the first-order condition for labor supply. Using that result in (42), we obtain the labor supply as stated in the Proposition, which we can in turn use in the labor supply optimality condition as in (41) to back out the stated consumption schedule.

A.2 Proof of Proposition 2

Using that $1 - N_{ss}^i = \gamma \frac{c_{ss}^i}{w_{ss} z^i}$ according to (9) and (10), we can derive the continuation value of a household that enters period 1 with b_1 bonds and productivity draw $i \in \{h, l\}$ to be

$$V_1^i(b_0) = \frac{1}{1-\beta} \left[(1+\gamma) \log \left(\frac{1}{1+\gamma} \left(w_{ss} z_i + \frac{i_{ss}}{1+i_{ss}} \left((1+i_1) b_0 - z^i \tau_1 \right) \right) \right) + \gamma \log \frac{\gamma}{w_{ss} z_i} \right]$$

This allows us to state the problem of a household in period 0 as

$$\max_{b_0, N_0} \left\{ \log(w_0 N_0 + T_0 - b_0) + \gamma \log(1 - N_0) + \beta \left[\rho^h V_1^h(b_0) + (1-\rho^h) V_1^l(b_0) \right] \right\}$$

which has the following first order conditions:

$$\begin{aligned} N_0 &: \quad \frac{w_0}{w_0 N_0 + T_0 - b_0} = \frac{\gamma}{1 - N_0} \\ b_0 &: \quad \frac{1}{w_0 N_0 + T_0 - b_0} = \beta \rho^h \frac{dV_1^h(b_0)}{db_0} + \beta(1-\rho^h)\frac{dV_1^l(b_0)}{db_0} \\ &= \frac{\beta}{1-\beta}\rho^h \frac{\frac{i_{ss}}{1+i_{ss}}(1+i_1)}{\frac{1}{1+\gamma}\left(w_{ss}z^h + \frac{i_{ss}}{1+i_{ss}}\left((1+i_1)b_0 - z^h\tau_1\right)\right)} \\ &+ \frac{\beta}{1-\beta}\rho^h \frac{\frac{i_{ss}}{1+i_{ss}}(1+i_1)}{\frac{1}{1+\gamma}\left(w_{ss}z^l + \frac{i_{ss}}{1+i_{ss}}\left((1+i_1)b_0 - z^l\tau_1\right)\right)} \end{aligned}$$

Since households are identical in period 0, they will make the same choices regarding bond-holdings. Thus, asset market clearing requires $b_0 = b_0^g$ and in turn $b_0 = T_0$ by the fiscal authority's budget constraint. Using that in the first order condition for labor supply, we obtain $N_0 = \frac{1}{1+\gamma}$. Since $c_0 = w_0 N_0 + T_0 - b_0$, it further follows that $c_0 = \frac{w_0}{1+\gamma}$. Substituting these results into the first order condition for bond holdings, it simplifies to

$$\begin{aligned} \frac{1}{w_0} &= \beta\rho^h \frac{(1+i_1)}{\left(w_{ss}z^h + \frac{i_{ss}}{1+i_{ss}}\left((1+i_1)b_0 - z^h\tau_1\right)\right)} \\ &+ \beta(1-\rho^h)\frac{(1+i_1)}{\left(w_{ss}z^l + \frac{i_{ss}}{1+i_{ss}}\left((1+i_1)b_0 - z^l\tau_1\right)\right)} \end{aligned} \tag{46}$$

which furthermore uses that $\frac{i_{ss}}{1+i_{ss}} = 1 - \beta$ due to the assumptions made above. Since we will have $\pi_1 = 1$ according to Proposition (1), it follows immediately from the Phillips curve (5) that

$$w_0 = \frac{\phi(\pi_0 - 1)\pi_0 + \epsilon - 1}{\epsilon} \quad . \tag{47}$$

Additionally, because of the assumed government policies, the tax rate in $t = 1$ needs to fulfil

$$\rho^h z^h \tau_1 + (1-\rho^h) z^l \tau_1 = \frac{1+i_1}{\pi_1} b_0^g \Leftrightarrow \tau_1 = (1+i_1) b_0^g$$

where the second step uses (1) and $\pi_1 = 1$. By substituting the above as well (47) and the Taylor rule (6) into (46), we obtain the equation that characterizes π_0 as stated in the Proposition:

$$\begin{aligned} \frac{\epsilon}{\epsilon - 1 + \phi(\pi_t - 1)\pi_t} &= \beta\rho^h \frac{1 + r_0^* + \theta_\pi(\pi_t - 1)}{w_{ss}z^h + \frac{i_{ss}}{1+i_{ss}}\left(1 + r_0^* + \theta_\pi(\pi_t - 1)\right) b_0^g (1 - z_h)} \\ &+ \beta(1-\rho^h)\frac{1 + r_0^* + \theta_\pi(\pi_t - 1)}{w_{ss}z^l + \frac{i_{ss}}{1+i_{ss}}\left(1 + r_0^* + \theta_\pi(\pi_t - 1)\right) b_0^g (1 - z^l)} \quad . \end{aligned}$$

A.3 Proof of Proposition 3

By the Implicit Function Theorem, since $r_0^n(g_b^0)$ is implicitly defined by $F(g_0^b, r_0^n(g_b^0)) = 0$ with

$$F(g_0^b, r_0^n) = \beta\rho^h \frac{1 + r_0^n}{w_{ss}z^h + \frac{i_{ss}}{1+i_{ss}}\left(1 + r_0^n\right) b_0^g(1 - z^h)} + \beta(1 - \rho^h)\frac{1 + r_0^n}{w_{ss}z^l + \frac{i_{ss}}{1+i_{ss}}\left(1 + r_0^n\right) b_0^g(1 - z^l)} - \frac{\epsilon}{\epsilon - 1}$$

we have

$$\frac{\partial r_0^n}{\partial b_g^0} = -\left. \frac{\partial F}{\partial b_g^0} \middle/ \frac{\partial F}{\partial r_0^n} \right. . \tag{48}$$

The derivatives in (48) are given by

$$\frac{\partial F}{\partial b_g^0} = \beta\rho^h \frac{\frac{i_{ss}}{1+i_{ss}}(1 + r_0^n)^2(z^h - 1)}{\left(w_{ss}z^h + \frac{i_{ss}}{1+i_{ss}}\left(1 + r_0^n\right) b_0^g(1 - z^h)\right)^2} + \beta(1 - \rho^h)\frac{\frac{i_{ss}}{1+i_{ss}}(1 + r_0^n)^2(z^l - 1)}{\left(w_{ss}z^h + \frac{i_{ss}}{1+i_{ss}}\left(1 + r_0^n\right) b_0^g(1 - z^l)\right)^2} \tag{49}$$

and

$$\frac{\partial F}{\partial r_0^n} = \beta\rho^h \frac{w_{ss}z^h}{\left(w_{ss}z^h + \frac{i_{ss}}{1+i_{ss}}\left(1 + r_0^n\right) b_0^g(1 - z^h)\right)^2} + \beta(1 - \rho^h)\frac{w_{ss}z^l}{\left(w_{ss}z^l + \frac{i_{ss}}{1+i_{ss}}\left(1 + r_0^n\right) b_0^g(1 - z^l)\right)^2} . \tag{50}$$

Since $w_{ss}z^h > 0$ and $w_{ss}z^l > 0$, it is clear that $\frac{\partial F}{\partial r_0^n} > 0$. However, things are less obvious for (49), as its first term is positive and its second term is negative since $z^l < 1$. However, notice that if

$$\underbrace{w_{ss}z^h + \frac{i_{ss}}{1 + i_{ss}}\left(1 + r_0^n\right) b_0^g(1 - z^h)}_{=(1+\gamma)c_{ss}^h} > \underbrace{w_{ss}z^l + \frac{i_{ss}}{1 + i_{ss}}\left(1 + r_0^n\right) b_0^g(1 - z^l)}_{=(1+\gamma)c_{ss}^l} \tag{51}$$

then

$$\begin{aligned}\frac{\partial F}{\partial b_g^0} &= \beta\rho^h \frac{\frac{i_{ss}}{1+i_{ss}}(1 + r_0^n)^2(z^h - 1)}{((1 + \gamma)c_{ss}^h)^2} + \beta(1 - \rho^h)\frac{\frac{i_{ss}}{1+i_{ss}}(1 + r_0^n)^2(z^l - 1)}{((1 + \gamma)c_{ss}^l)^2} \\ &< \beta\rho^h \frac{\frac{i_{ss}}{1+i_{ss}}(1 + r_0^n)^2(z^h - 1)}{((1 + \gamma)c_{ss}^h)^2} + \beta(1 - \rho^h)\frac{\frac{i_{ss}}{1+i_{ss}}(1 + r_0^n)^2(z^l - 1)}{((1 + \gamma)c_{ss}^h)^2} = 0\end{aligned}$$

(the final equality follows from (1)). This means that if (51) holds, then $\frac{\partial F}{\partial b_g^0} < 0$ and thus $\frac{\partial r_0^n}{\partial b_g^0} > 0$. For that to be the case, we require

$$w_{ss} - \frac{i_{ss}}{1 + i_{ss}}(1 + r_0^n(b_0^g))b_g^0 > 0 . \tag{52}$$

Under our restriction $0 \leq b_0^g < \frac{\epsilon-1}{\epsilon}\frac{\beta}{1-\beta} = \frac{w_{ss}}{i_{ss}}$, this will be the case. Notice first that $r_0^n(b_0^g)$ as implicitly defined by (13) fulfills

$$r_0^n(0) = \frac{1}{\beta\left(\frac{\rho^h}{z^h} + \frac{1-\rho^h}{z^l}\right)} - 1 < \frac{1}{\beta} - 1 = r_0^n\left(\frac{w_{ss}}{i_{ss}}\right) = i_{ss} \tag{53}$$

as $\frac{\rho^h}{z^h} + \frac{1-\rho^h}{z^l} > \frac{1}{\rho^h z^h + (1-\rho^h)z^l} = 1$ due to Jensen's Inequality. Now, guess that (52) does hold for any $b_0^g \in [0, w_{ss}/i_{ss})$. In that case, our previous results imply that r_0^n is increasing at any of these values. But then, as there is no discontinuity around w_{ss}/i_{ss}, (53) requires that we also must have $r_0^n(b_0^g) < i_{ss}$ at these values. In turn, (52) is indeed true and we have $c_{ss}^h > c_{ss}^l$. So, it is also clear that $\frac{\partial r_0^n}{\partial b_g^0} < 0$, establishing the proposition that $\frac{\partial r_0^n}{\partial b_g^0} > 0$.

A.4 Proof of Proposition 4

We proceed similar as in Appendix (A.3). Given that (12) implicitly defines π_0 as a function of b_0^g for given parameters,

$$\frac{\partial \pi_0}{\partial b_g^0} = -\left.\frac{\partial F^\pi}{\partial b_g^0}\right/\frac{\partial F^\pi}{\partial \pi_0} \quad . \tag{54}$$

with

$$\begin{aligned} F^\pi(g_0^b, \pi_0) = \beta\rho^h &\frac{1 + r_0^* + \theta_\pi(\pi_0 - 1)}{w_{ss}z^h + \frac{i_{ss}}{1+i_{ss}}\left(1 + r_0^* + \theta_\pi(\pi_0 - 1)\right) b_0^g(1 - z_h)} \\ &+ \beta(1-\rho^h)\frac{1 + r_0^* + \theta_\pi(\pi_0 - 1)}{w_{ss}z^l + \frac{i_{ss}}{1+i_{ss}}\left(1 + r_0^* + \theta_\pi(\pi_0 - 1)\right) b_0^g(1 - z^l)} - \frac{\epsilon}{\epsilon - 1 + \phi(\pi_0 - 1)\pi_0} \end{aligned}$$

Similarly to (49), we have

$$\begin{aligned} \frac{\partial F^\pi}{\partial b_g^0} = \beta\rho^h &\frac{\frac{i_{ss}}{1+i_{ss}}(1 + r_0^* + \theta_\pi(\pi_0 - 1))^2(z^h - 1)}{\left(w_{ss}z^h + \frac{i_{ss}}{1+i_{ss}}\left(1 + r_0^* + \theta_\pi(\pi_0 - 1)\right) b_0^g(1 - z^h)\right)^2} \\ &+ \beta(1-\rho^h)\frac{\frac{i_{ss}}{1+i_{ss}}(1 + r_0^* + \theta_\pi(\pi_0 - 1))^2(z^l - 1)}{\left(w_{ss}z^h + \frac{i_{ss}}{1+i_{ss}}\left(1 + r_0^* + \theta_\pi(\pi_0 - 1)\right) b_0^g(1 - z^l)\right)^2} \quad . \end{aligned} \tag{55}$$

Additionally,

$$\frac{\partial F^\pi}{\partial \pi_0} = \beta\rho^h\frac{\theta_\pi w_s s z^h}{(1+\gamma)c_{ss}^h} + \beta(1-\rho^h)\frac{\theta_\pi w_s s z^l}{(1+\gamma)c_{ss}^l} + \frac{\epsilon\phi(2\pi_0 - 1)}{(\epsilon - 1 + \phi(\pi_0 - 1)\pi_0)^2} \quad .$$

Since we are considering the derivative around $r^* = r_0^n(b_g^0)$ at which $\pi_0 = 1$, $2\pi_0 - 1 > 0$ so all the terms are positive and $\frac{\partial F^\pi}{\partial \pi_0} > 0$.

Regarding (55), we can use the same argument as in Appendix (A.3). If $c_{ss}^h > c_{ss}^l$, then

$$\begin{aligned} \frac{\partial F^\pi}{\partial b_g^0} &= \beta\rho^h\frac{\frac{i_{ss}}{1+i_{ss}}(1 + r_0^* + \theta_\pi(\pi_0 - 1))^2(z^h - 1)}{((1+\gamma c_{ss}^h))^2} + \beta(1-\rho^h)\frac{\frac{i_{ss}}{1+i_{ss}}(1 + r_0^* + \theta_\pi(\pi_0 - 1))^2(z^l - 1)}{((1+\gamma c_{ss}^l))^2} \\ &< \beta\rho^h\frac{\frac{i_{ss}}{1+i_{ss}}(1 + r_0^* + \theta_\pi(\pi_0 - 1))^2(z^h - 1)}{((1+\gamma c_{ss}^h))^2} + \beta(1-\rho^h)\frac{\frac{i_{ss}}{1+i_{ss}}(1 + r_0^* + \theta_\pi(\pi_0 - 1))^2(z^l - 1)}{((1+\gamma c_{ss}^h))^2} = 0 \end{aligned}$$

Again, we are considering the case with $r_0^* = r_0^n(b_g^0)$ for some $b_g^0 \in [0, w_{ss}/i_{ss})$, so from the analysis in Appendix A.3, we know that indeed $c_{ss}^h > c_{ss}^l$ in this case. Thus $\frac{\partial F^\pi}{\partial b_g^0} < 0$ and hence $\frac{\partial \pi_0}{\partial b_g^0} > 0$, which establishes the proposition.

B Details on Section 4

B.1 Derivation of the wage Phillips curve

Given the uniform hours $N_{it} = N_{ut}$ for all union members and demand schedule (30), we have

$$\frac{\partial}{\partial W_{ut}} N_{ut} = -\epsilon_w \frac{N_{ut}}{W_{ut}}$$

and

$$\frac{\partial}{\partial W_{ut}} u(c_{it}) = u'(c_{it})(1-\tau_p)(1-\tau_w)\left(s_{it}\frac{W_{ut}}{P_t}N_{ut}\right)^{-\tau_p} s_{it}\frac{N_{ut}}{P_t}(1-\epsilon_w) \quad ,$$

the latter reflecting that due to the envelope theorem, the marginal utility of additional resources should equal the marginal utility of consumption. In turn, the F.O.C. corresponding to (31) is

$$(1-\tau_p)(1-\tau_w)\frac{1-\epsilon_w}{W_{ut}}\int\left(u'(c_{it})\left(s_{it}\frac{W_{ut}}{P_t}N_{ut}\right)^{1-\tau_p}\right)di - \frac{\epsilon_w}{W_{ut}}\varsigma N_{ut}^{1+\gamma}$$
$$-\psi\left(\frac{W_{ut}}{W_{ut-1}}-1\right)\frac{1}{W_{ut-1}} + \beta\mathbb{E}_t\psi\left(\frac{W_{ut+1}}{W_{ut}}-1\right)\frac{W_{ut+1}}{(W_{ut})^2} = 0 \quad .$$

If we now use that unions are symmetric and thus $N_{ut} = N_t$ and $W_{ut} = W_t$, re-arranging yields

$$\pi_t^w(1-\pi_t^w) = \frac{\epsilon_w}{\psi}\left(\frac{\epsilon_w-1}{\epsilon_w}(1-\tau_p)(1-\tau_w)\int\left(u'(c_{it})\left(s_{it}\frac{W_t}{P_t}N_t\right)^{1-\tau_p}\right)di - \varsigma N_t^{1+\gamma}\right)$$
$$+\beta\mathbb{E}_t\pi_{t+1}^w(1-\pi_{t+1}^w) \quad .$$

B.2 Definition of equilibrium

Definition 1. *A* ***Recursive Equilibrium*** *of the model consists of*

- *value functions* $V^a(a_{it}, k_{it}, s_{it}, \Psi_{it}; \Gamma_t)$, $V^{na}(a_{it}, k_{it}, s_{it}, \Psi_{it}; \Gamma_t)$
- *household policies* $a^a(a_{it}, k_{it}, s_{it}, \Psi_{it}; \Gamma_t)$, $a^{na}(a_{it}, k_{it}, s_{it}, \Psi_{it}; \Gamma_t)$, $k(a_{it}, k_{it}, s_{it}, \Psi_{it}; \Gamma_t)$ *and* $c^a(a_{it}, k_{it}, s_{it}, \Psi_{it}; \Gamma_t)$, $c^{na}(a_{it}, k_{it}, s_{it}, \Psi_{it}; \Gamma_t)$,
- *firm- and union policies* I_t, K_t, H_t, Y_t, u_t, B_t^l, Π_t, $y_{jt}\forall j \in [0,1]$, w_t
- *prices* h_t, r_t, q_t, r_t^l, mc_t
- *inflation* π_t
- *government policies* $G_t, B_{t+1}^g, \tau_t, r_{t+1}^R$,

so that

1. *Given prices* r_t^l, r_t, q_t, *wages* w_t *and profits* Π_t, *the value functions* $V^a(a_{it}, k_{it}, e_{it}, s_{it}, \Xi_{it}; \Gamma_t)$, $V^{na}(a_{it}, k_{it}, e_{it}, s_{it}, \Xi_{it}; \Gamma_t)$ *solve the households' Bellman equations in* (22) *and* (23) *and* $a(a_{it}, k_{it}, e_{it}, s_{it}, \Xi_{it}; \Gamma_t)$, $k(a_{it}, k_{it}, e_{it}, s_{it}, \Xi_{it}; \Gamma_t)$, $c(a_{it}, k_{it}, e_{it}, s_{it}, \Xi_{it}; \Gamma_t)$ *are the resulting optimal policy functions.*

2. $y_{jt} \in [0, 1]$ *are consistent with demand schedule* (4) *and final output* Y_t *given by* (3).

3. *Inflation* π_t *is consistent with Phillips curve* (24).

4. *Given prices* h_t, r_t, q_t, mc_t *and technology shock* Z_t *the intermediate goods producers choices* K_t, H_t, u_t *are consistent with optimality conditions* (26)-(28).

5. *Given price* q_t, *the intermediate goods producers choices* I_t *are consistent with optimality condition* (29).

6. *The labor packer's zero profit condition* $h_t = w_t$ *is fulfilled.*

7. *The wage level* w_t *is consistent with* (32).

8. *The Liquid Asset Funds' portfolio choice is consistent with* (37).

9. *The return of liquid assets is given by* (38).

10. *Given inflation* π_t *and output growth* Y_t/Y_{t-1}, *the monetary authority set* r_{t+1}^R *according to* (33).

11. *Taking the remaining values as given, the government sets taxes according to* (35) *and issues debt* B_{t+1}^g *so that* (34) *holds.*

12. *The market for liquid asset clears, i.e.*

$$A_t^l = \int_0^1 a_{it} di \quad .$$

13. *The government bond market clears, i.e.*

$$B_t^l = B_t^g \quad .$$

14. *Capital market clearing requires, i.e.*

$$K_t = \frac{A_t^l - B_t^l}{q_{t-1}} + \int_0^1 k_{it} di \quad .$$

15. *The market for investment good clears, i.e.*

$$K_{t+1} = (1 - \delta(u_t))K_t + \left[1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t$$

16. *The market for labor services clears, i.e.*

$$H_t = N_t \int_{m^\Xi}^1 s_{it} di.$$

17. *The market for intermediate goods clears, i.e.*

$$\int_0^1 y_t(j)dj = F_t(u_t K_t, H_t)$$

18. *The final good market clears, i.e.*

$$\begin{aligned} Y_t =& C_t + G_t + I_t + \frac{\phi}{2}\left[\frac{I_t}{I_{t-1}} - 1\right]^2 + \bar{R}\int_0^1 \mathbb{1}(a_{it} \leq 0)a_{it}di \\ & - \left[\phi + \frac{\Psi}{2}\left(1 - \frac{B_t^l}{A_t^l}\right)^2\right] A_t^l \end{aligned}$$

where $\mathbb{1}(\cdot)$ *denotes the indicator function.*

B.3 Details on numerical implementation

B.3.1 Details on Steady State Solution

The household problem needs to be solved on a discretization of the state space: I choose 90 grid points for both a and k, either of which are non-linearily spaced as household decision functions tend to be more non-linear for lower levels of assets. In particular, the grid points for both a for k are spaced according to the "double exponential" rule, i.e.

$$\mathcal{X} = x_{min} + \exp(\exp(\mathbf{u}(\log(1 + \log(1 + x_{max} - x_{min})), n_i)) - 1) - 1$$

where x_{min} is the minimum value on the grid for variable x, x_{max} the maximum value and $\mathbf{u}(0, x_{max})$ a vector of equidistant points on the interval $[0, x_{max}]$. Since household value- and policy functions will feature and additional kink around $a = 0$ when the borrowing penalty kicks in, I add 5 additional grid points in the immediate vicinity of that point. Given that individual labor productivity is discretized to 17 points, this means that the household problem is solved on a tensor grid of $90 \times 90 \times (17 + 1) = 145800$ points (the "entrepreneur" status adds an "income" state to the 17 'skill" states). The discretization of the individual labor productivity process follows an off-the-shelf method à la Tauchen (1986). Whenever interpolation is needed off the grid, I use linear interpolation
For the implementation of the multidimensional EGM algorithm, I follow the replication codes for Bayer et al. (2024) closely.[33] Given the random illiquid asset adjustment, the EGM scheme only iterates over marginal value functions (i.e. the derivatives of V with respect to m and k) and does not compute V directly.
For finding the steady, I iterate over r_{ss}^l and r_{ss}^k: Given these values, the remaining steady variables can be backed out and the household-problem solved. I then use a heuristic updating procedure to search for r_{ss}^l and r_{ss}^k so that the asset markets clear.

[33] As of October 2024, these replication codes are available under https://github.com/BASEforHANK/BASEtoolbox.jl.

B.3.2 Details on State Space Perturbation

As already indicated in the main text, the model's dynamic equilibrium is approximated using First-Order Perturbation around its non-stochastic steady state.
For the State Space perturbation à la Bayer et al. (2024), note that when using the discretized representations of the marginal value functions as well as the joint income/asset distribution, the equilibrium can be represented as the solution to a non-linear difference equation of the form

$$\mathbb{E}_t F(\mathbf{y_t}, \mathbf{x_t}, \mathbf{y_{t+1}}, \mathbf{x_{t+1}}) = 0 \tag{56}$$

as e.g. used by Schmitt-Grohe and Uribe (2004). $\mathbf{y}$ denotes a vector of control variables, which includes the households' marginal value functions on the grid and $\mathbf{x}$ a vector of state variables, which includes the discretized distribution.
In theory, one could find the linearized equilibrium using the standard approach of computing the Jacobians of F as in (56) and then solve the resulting linear difference equation relying on methods such as Klein (2000). In practice, however, such an approach would involve very high computational costs for the 2-asset HANK model, given that the full $\mathbf{y}$ and $\mathbf{x}$ have a combined length exceeding $200,000$.
To overcome this problem, Bayer et al. (2024) propose a procedure which conducts dimension reduction in 2 steps, one before computing the Jacobians and one after. Specifically, it first uses a Discrete Cosine Transform (DCT) to dimension-reduce the marginal value functions: The values resulting from such a DCT are coefficients of a fitted multidimensional Chebychev polynomial, of which only a subset are selected to be perturbed: Bayer et al. (2024) propose to use the nodes that are most important for describing the derivatives of the steady state marginal value functions to changes in the set of prices that households directly take into account (e.g., interest rates and the wage). The other coefficients are kept at their steady state values.
For reducing the dimensionality of the joint distribution in the first step, the authors furthermore suggest splitting it into marginals and a copula, where the latter is in effect treated as an interpolator mapping the steady state marginal distributions into the joint distribution. That "interpolator" can also be dimension-reduced through a DCT or just kept fixed, so one only perturbs the marginals as well as selected coefficients of the copula, which have substantially lower dimension. Overall, in my application the procedure manages to shrink the effective dimensionality of the system to a manageable number of approx. 1400, for which an initial perturbation solution is obtained.
The second step further reduces the set of DCT coefficients by using the first step solution to check which ones vary only very little with the aggregate shocks and are thus not important for explaining model dynamics. It is useful if the model has to be repeatedly solved for different parameters, such as for checking model stability for different parameters as e.g., in Section 6. For a more detailed exposition, see Bayer et al. (2024).

B.3.3 Details on Sequence Space Perturbation

As already mentioned in the main text, in addition to the State Space perturbation method described above, I also use a Sequence Space linearization method à la Auclert et al. (2021), as its allows to flexibly expose the economy to various news shocks. This proves useful particular for the analysis in Section 9: Firstly, the analysis in that Section requires the model to be able to handle a binding lower bound on the nominal interest rate. As already pointed out by McKay and Wieland (2021), this can be achieved (relatively) easily in a Sequence space setting using news shocks. The idea is that if an aggregate shock would cause the central bank (CB) to violate the ELB in a certain number of periods, one can solve for a sequence of pre-announced monetary policy (news) shocks that would keep the economy at the ELB instead. The CB then enforces the ELB by announcing exactly these shocks. Secondly, the same feature of a sequence space solution makes it easy to consider different interest rules. This is because in a linearized model, those can similarly be imposed by announcing a suitable set of news shocks to the policy rule in place (McKay and Wolf, 2023).

I obtain the Sequence Space Jacobians (SSJs) of the model's heterogeneous agent block as described in Auclert et al. (2021), although I rely on automatic differentiation instead of finite differences to ensure accuracy of the derivatives. For obtaining the general equilibrium SSJ's, I then build on the model representation proposed by Bhandari et al. (2023), which, for given Heterogeneous Agents SSJs, yields the GE SSJs as Auclert et al. (2021)'s Directed Acyclical Graph (DAG) approach.[34] I use a truncation horizon of $T = 500$ periods, as my 2-asset HANK model features relatively-long lived IRFs due to the presence of investment adjustment costs and shocks with quite high persistence.

B.3.4 Details on Filtering algorithm

To construct series of business cycle shocks making the model match given time series of observed variables, I adopt the filtering method developed by McKay and Wieland (2021). As these authors show, it can be interpreted as a restricted version of a Kalman filter. A brief description of the approach is provided below:

The method is applicable if we have a vector Y_t of n_y observed variables for a number of periods $t = 1, .., T_{obs}$ and want to obtain vectors of n_e shocks ϵ_t which make the linearized model generate the outcomes Y_t. This requires $n_e \geq n_y$, with my description below focussing on the case $n_e = n_y$ relevant for my application. Let $R(\tau, i)$ denote vectors containing $(\tau + 1)$th elements of the IRFs for the variables in Y in response to a unit change in the ith element of ϵ_t, which can be obtained using one of the solution methods

[34] An example application for a simpler HANK model can be found under `https://mhaense1.github.io/SSJ_Julia_Notebook/SSJ_notebook_2.html`.

described above. The matrix

$$R_\tau = [R(\tau, 1), R(\tau, 2), ..., R(\tau, n_e)]$$

concatenating these vectors horizontally thus describes how the shock vector ϵ_t affects the observables in period $t + \tau$.

It is assumed that the model is initially in steady state and that $Y_0 = Y_{ss} = 0$. Denote by Q_t cumulative effect of previous shocks on Y_t, i.e.

$$Q_t = \sum_{\tau=0}^{t-1} R(t - \tau)\epsilon_\tau \tag{57}$$

Naturally, $Q_1 = Y_0$. One can then obtain obtain $\{\epsilon_t\}_{t=1}^{T_{obs}}$ as follows: Starting from $t = 1$, get ϵ_t as

$$\epsilon_t = R(0)^{-1}(Y_t - Q_t) \tag{58}$$

and then compute Q_{t+1} as in (57). Afterwards, do the same for $t + 1$ and so on.

A complication arises if the model features a potentially binding ELB on the policy rate r^R, as it indeed does in my application. While a SSJ solution allows simulating the model under an ELB by imposing it via news shocks to the monetary policy rule, a simultaneity problem arises for filtering: If the shocks as obtained in (58) would cause the model to eventually violate the ELB, adding additional news shocks will cause those ϵ_t to no longer produce the empirical Y_t. Hence, ϵ_t and the ELB news shocks need to solved for jointly, for which McKay and Wieland (2021) propose an iterative procedure that I adapt to my setting.

C Data Construction

C.1 Data for Local Projections

The data used for the Local Projection evidence in Section ?? is constructed as follows, with all data obtained from the Federal Reserve Economic Data (FRED) platform:

- **Public Debt**: The Market Value of Marketable Treasury Debt (MVMTD027MNFRBDAL) relative to GDP (GDP)
- **Inflation Expectation**: 5-Year Expected Inflation according to the Cleveland Fed's estimate (EXPINF5YR)
- **Expected 5-year Bond Return**: Market Yield on U.S. Treasury Securities at 5-Year Constant Maturity (DGS5) minus the 5-Year Inflation expectation (EXPINF5YR)
- **Inflation**: Annualized inflation rates computed from the GDP deflator (GDPDEF)

- **Government Expenditures**: Federal Government: Current Expenditures (FGEXPND) relative to current GDP
- **Unemployment**: Series UNRATE

C.2 Data for model exercise

This Appendix describes the construction of the data used for the model exercises in Section 9. The analysis uses the following aggregate variables, the data for which were again obtained from the Federal Reserve Economic Data (FRED) platform:

- **Nominal Rate**: The variable corresponds to the federal funds rate (FRED series FEDFUNDS).
- **Inflation**: (Gross) Inflation corresponds to the growth of the GDP deflator (GDPDEF) compared to the previous quarter.
- **Output**: (Real) Output corresponds to the sum of the following variables divided by the GDP deflator and current population (CNP16OV):
  - Personal Consumption Expenditures: Non-durable Goods (PCND)
  - Personal Consumption Expenditures: Durable Goods (PCDG)
  - Personal Consumption Expenditures: Services (PCESV)
  - Gross Private Domestic Investment (GPDI)
  - Government Consumption Expenditure and Gross Investment (GCE)
- **Investment**: Gross Private Domestic Investment (GPDI) divided by the GDP deflator and current population (CNP16OV).
- **Transfers**: (Real) Transfer payments consist of the sum of the following variables divided by the GDP deflator and current population (CNP16OV).
  - Federal government current transfer payments: Government social benefits to persons (B087RC1Q027SBEA)
  - Federal government current transfer payments: Grants-in-aid to state and local governments (FGSL)

  The definition of this variable follows Bianchi et al. (2023).
- **Hours worked:** Nonfarm Business Sector Hours Worked for All Workers (HOANBS) divided by either the level of the civilian labor force (CNP16OV) or the civilian labor force (CLF16oV).
- **Labor Compensation:** Compensation of Employees (W209RC1) divided by the GDP deflator (GDPDEF) and current population (CNP16OV).

Rate gap	δ_0	β	ζ	λ	$\bar{R}$	G_{ss}
3.74% (Baseline model)	0.0175	0.9838	0.0005	0.0363	0.0355	0.5650
2.71%	0.02	0.9866	0.0004	0.067	0.0299	0.5832
1.70%	0.0225	0.9894	0.0003	0.1068	0.0222	0.5986
0.69%	0.025	0.9921	0.0003	0.1760	0.0131	0.6137

Table D.1: Alternative calibrations used in Section 6

The pre-covid trends for Output, Government Consumption, Investment and Transfers are taken to be linear time trends estimated for the respective variables over the period 2014Q1 to 2019Q4.

Finally, for the comparison between model-implied and actual public debt, I use an approximation of the market value of treasury debt held by the domestic debt public: To the best of my knowledge, there is no publicly available breakdown of the market value of US treasury debt into domestic and foreign holdings throughout the entire period 2020Q1-2024Q2. Instead, I calculate a "foreign share" s^f as Federal Debt Held by Foreign and International Investors (FDHBFIN) over Federal Debt held by the Public (FYGFDPUN) and then take the market value of domestically-held public debt to be $(1 - s^f)$ times the Market Value of Marketable Treasury Debt (MVMTD027MNFRBDAL) minus Federal government checkable deposits and currency as reported in Federal Reserve's Financial Accounts of the United States (FL313020005.Q): The latter would reduce the governments net liquid asset supply from the perspective of the model.

Implicitly, the approximation being correct requires the treasury debt portfolios held by domestic and foreign agents to not differ systematically in terms of maturity structure etc. The US Treasury (2024) reports foreign holdings to have weighted average maturity of 6.3 years, a bit but not overwhelmingly higher than the overall average.

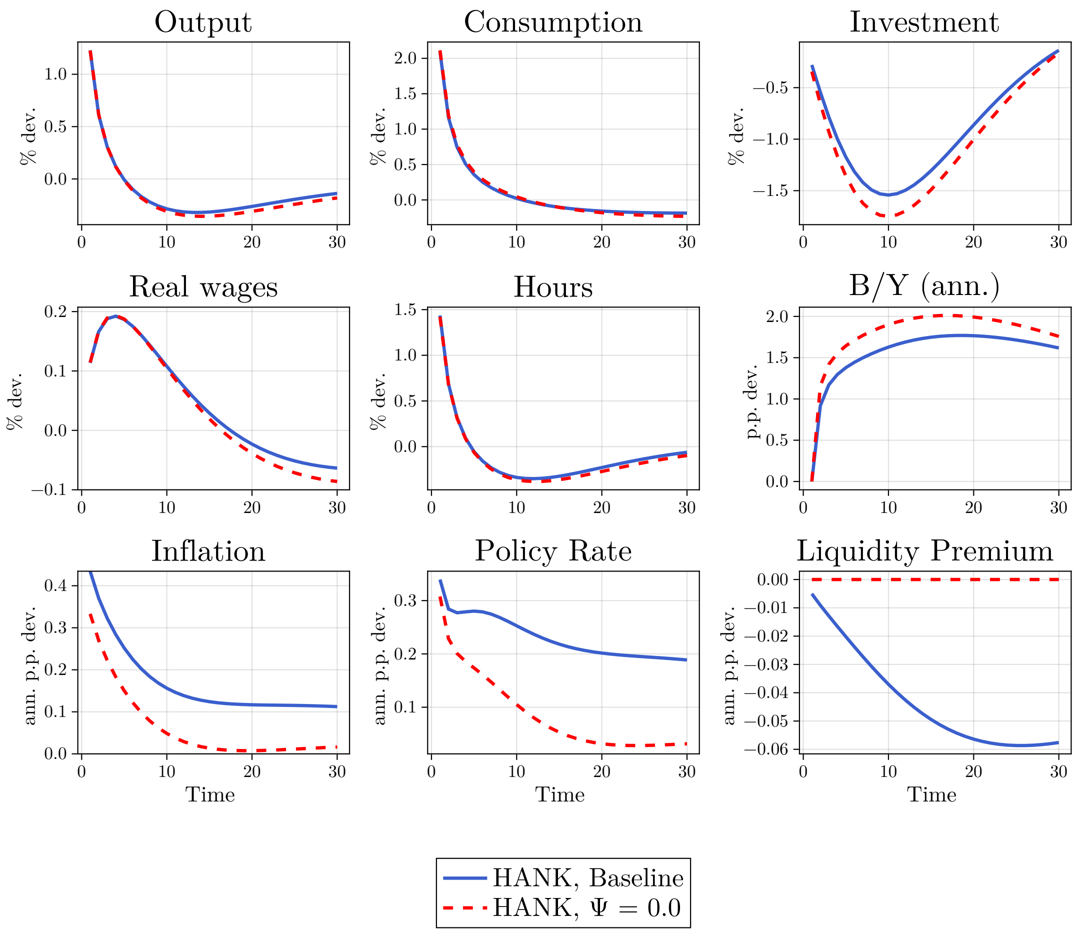
D Additional Tables

This Appendix contains auxiliary Tables referred to in the main text.

E Additional Figures

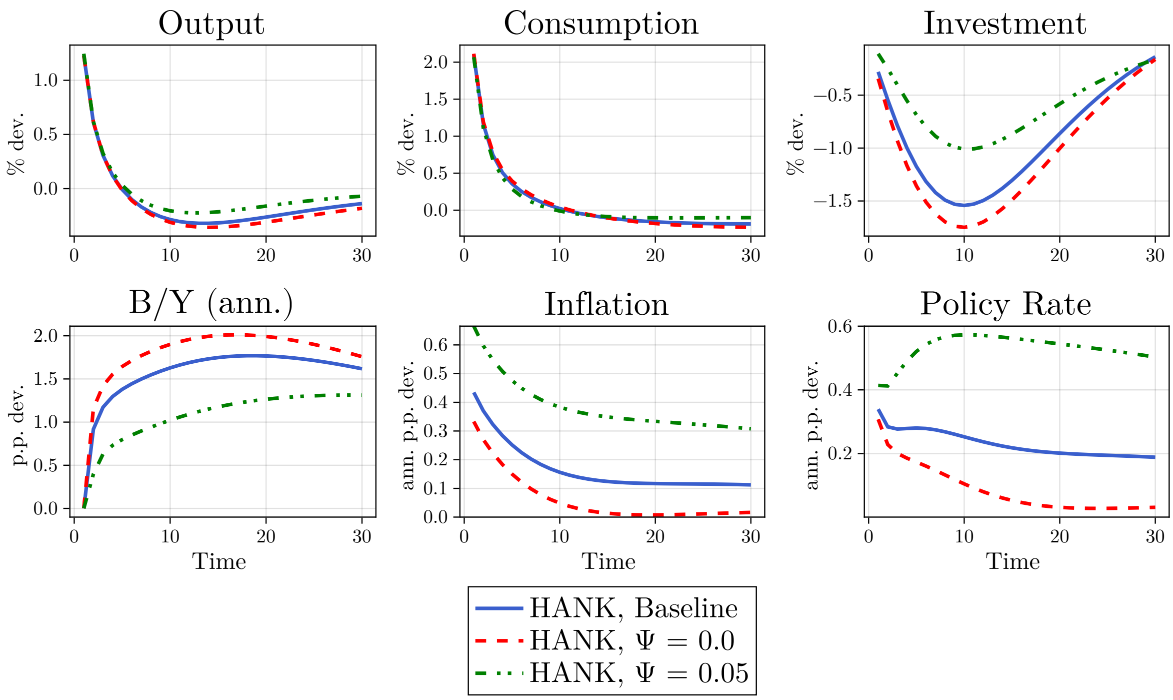
This Appendix contains auxiliary Figures referred to in the main text.

Figure E.1: Model IRFs to fiscal shock



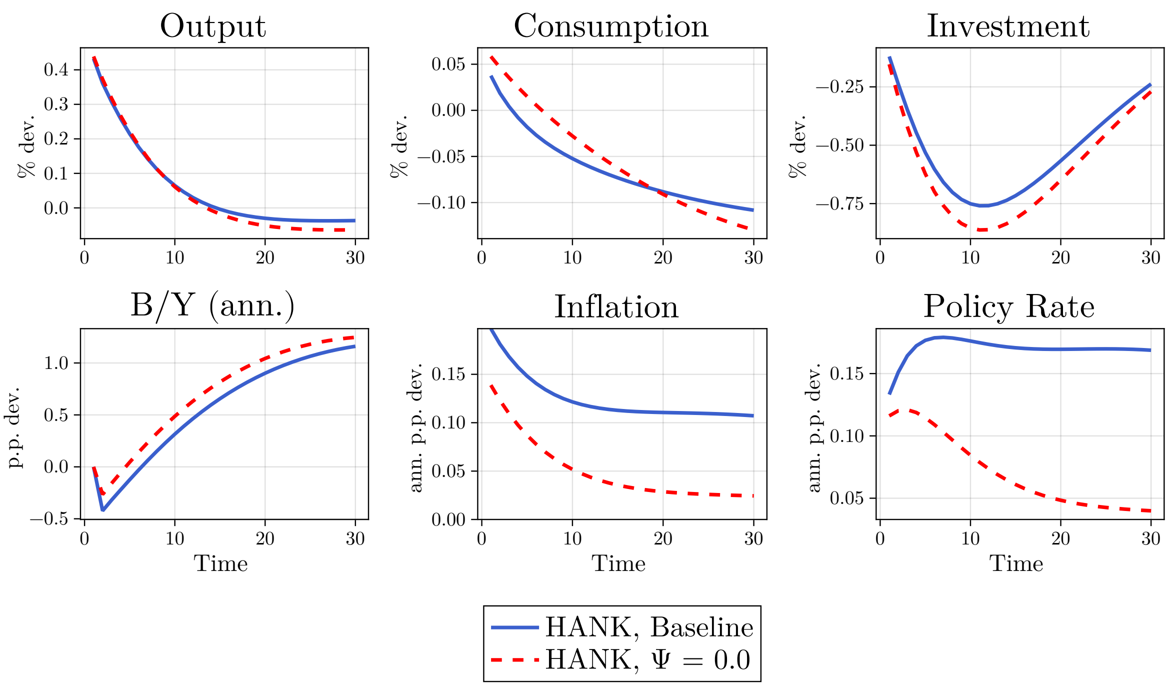
Note: B/Y represents the real market value of public debt B^g over 4 times (=annualized) GDP. The liquidity premium is defined as $\mathbb{E}_t\left(\frac{q_{t+1}+r^k_{t+1}}{q_t} - \frac{R^r_t}{\pi_{t+1}}\right)$.

Figure E.2: Model IRFs: More segmented markets



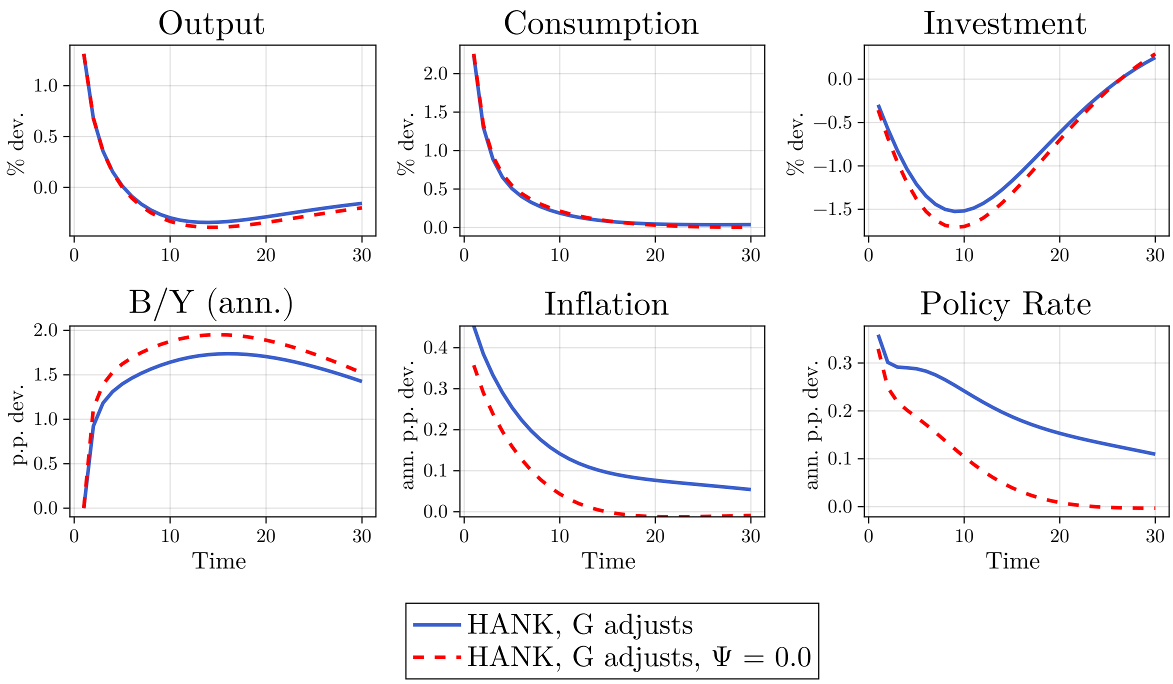
Note: B/Y represents the real market value of public debt B^g over annualized GDP. Figures display relative (in %) or percentage point (p.p.) deviations from Steady State.

Figure E.3: Model IRFs to Government spending shock



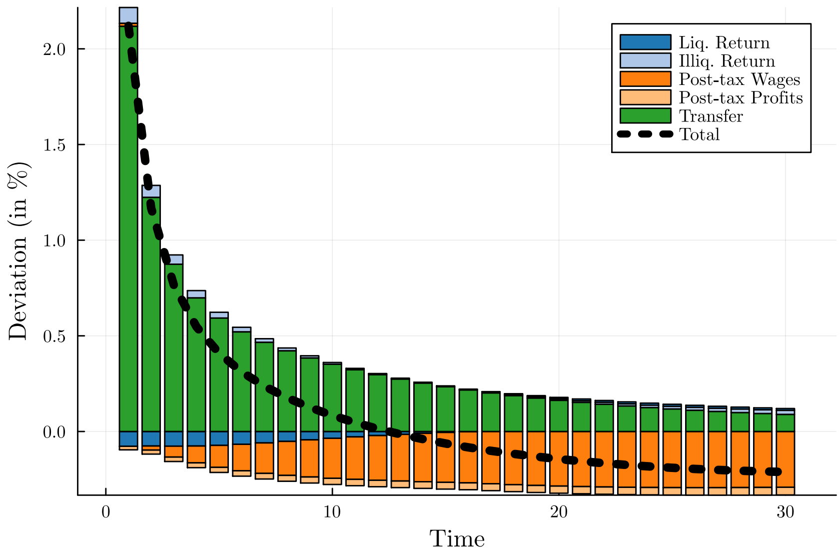
Note: B/Y represents the real market value of public debt B^g over annualized GDP. Figures display relative (in %) or percentage point (p.p.) deviations from Steady State.

Figure E.4: Model IRFs to Transfer shocks: G adjusts

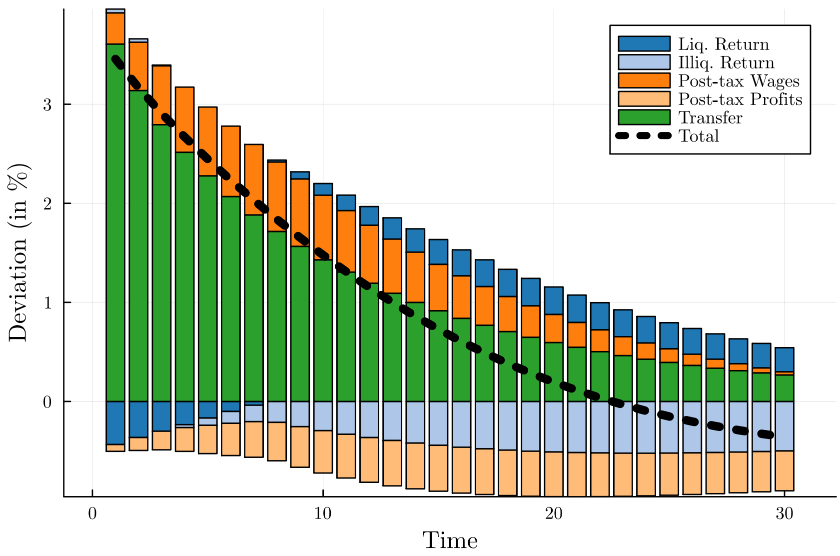


Note: B/Y represents the real market value of public debt B^g over annualized GDP. Figures display relative (in %) or percentage point (p.p.) deviations from Steady State. Instead of adjusting taxes, the government is assumed to consolidate its finance by following the rule $\log G_t = \rho_G \log G_{t-1} + (1 - \rho_G)(\log G_{SS} + \psi_B(\log B_t^g - \log B_{SS}))$ with $\rho_G = 0.94$.

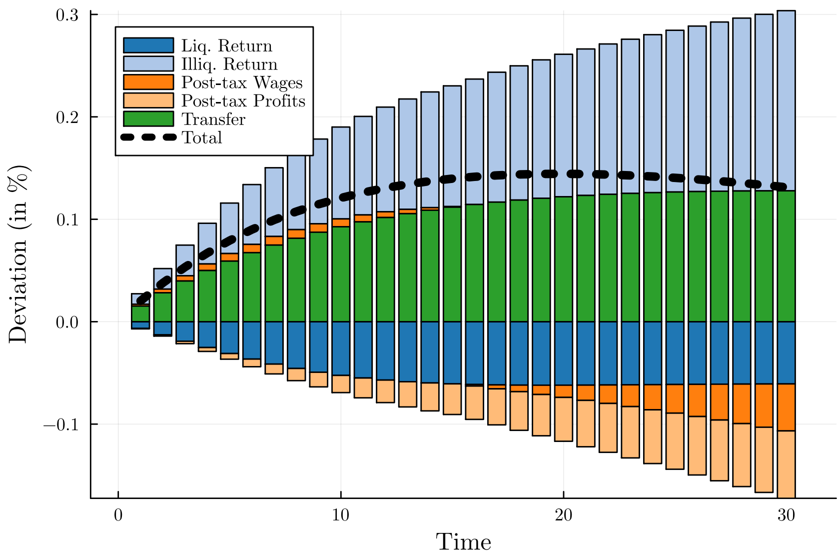
Figure E.5: Decomposition of household responses in $\Psi = 0$ economy



(a) Consumption Response

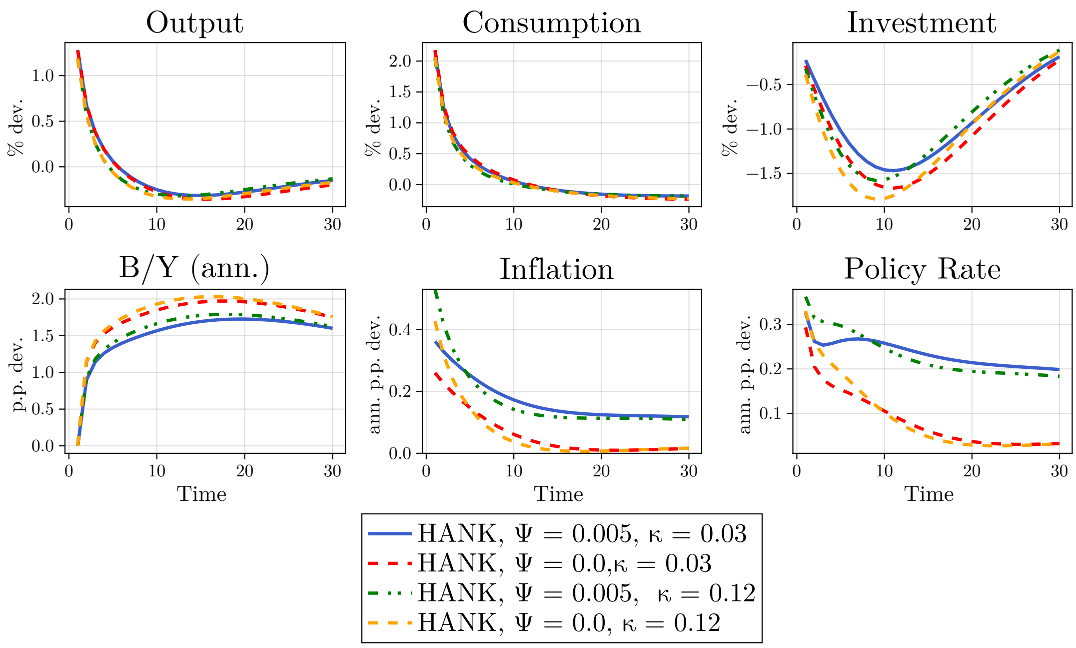


(b) Liquid Savings Response



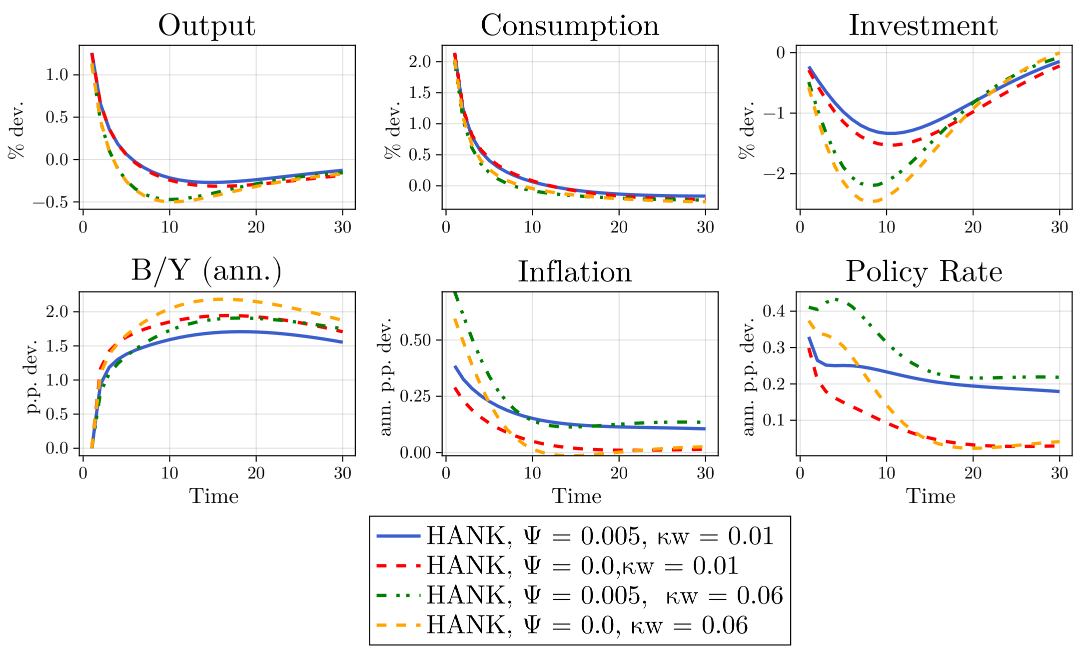
(c) Illiquid Savings Response

Figure E.6: IRFs to transfer shock: Varying κ



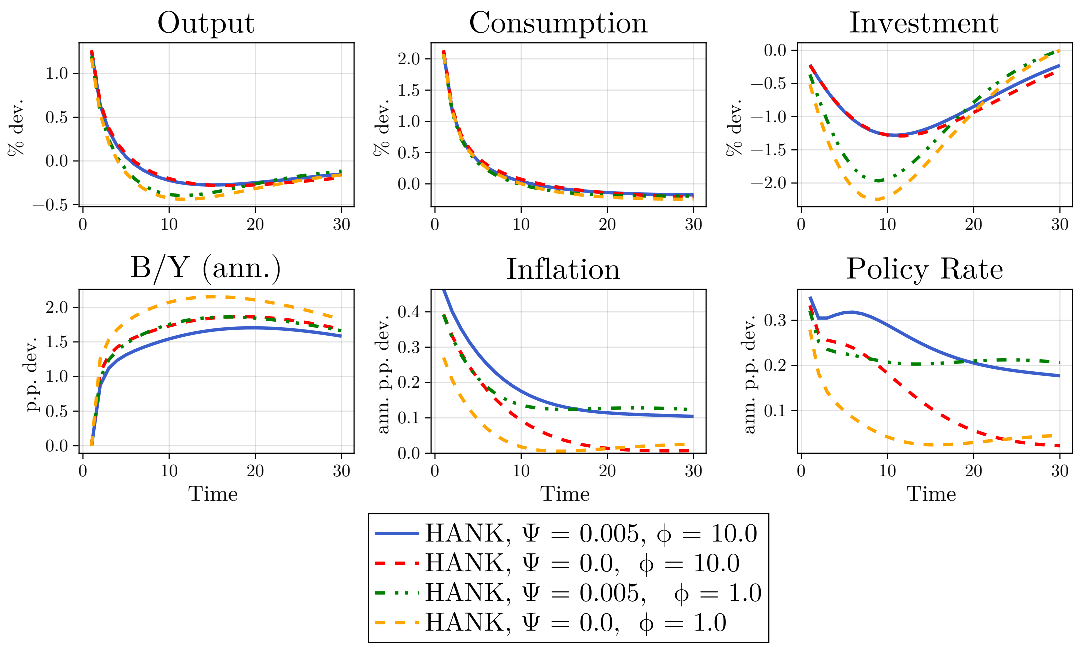
Note: B/Y represents the real market value of public debt B^g over 4 times (=annualized) GDP.

Figure E.7: IRFs to transfer shock: Varying κ_w



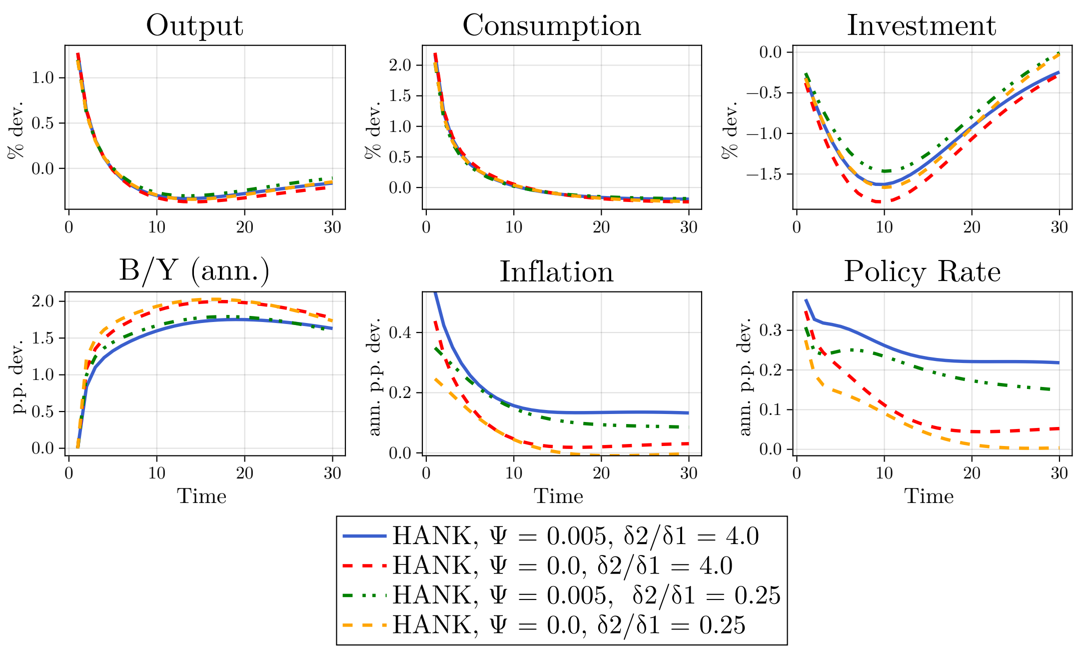
Note: B/Y represents the real market value of public debt B^g over 4 times (=annualized) GDP.

Figure E.8: IRFs to transfer shock: Varying ϕ



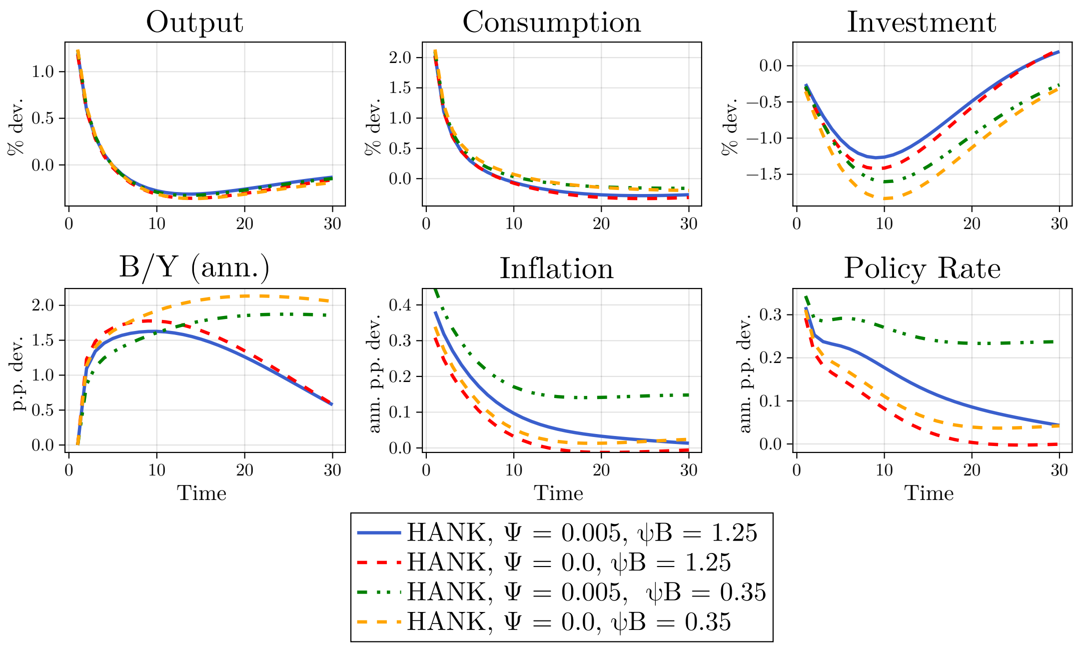
Note: B/Y represents the real market value of public debt B^g over 4 times (=annualized) GDP.

Figure E.9: IRFs to transfer shock: Varying δ_2/δ_1



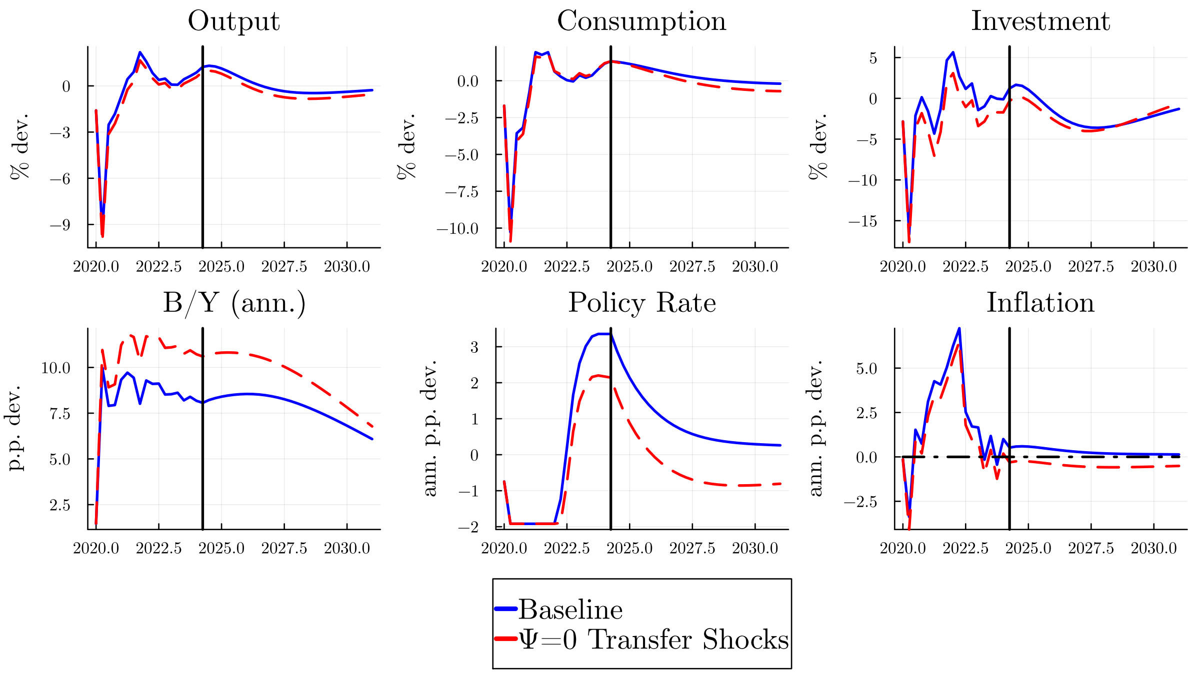
Note: B/Y represents the real market value of public debt B^g over 4 times (=annualized) GDP.

Figure E.10: IRFs to transfer shock: Varying ψ_B



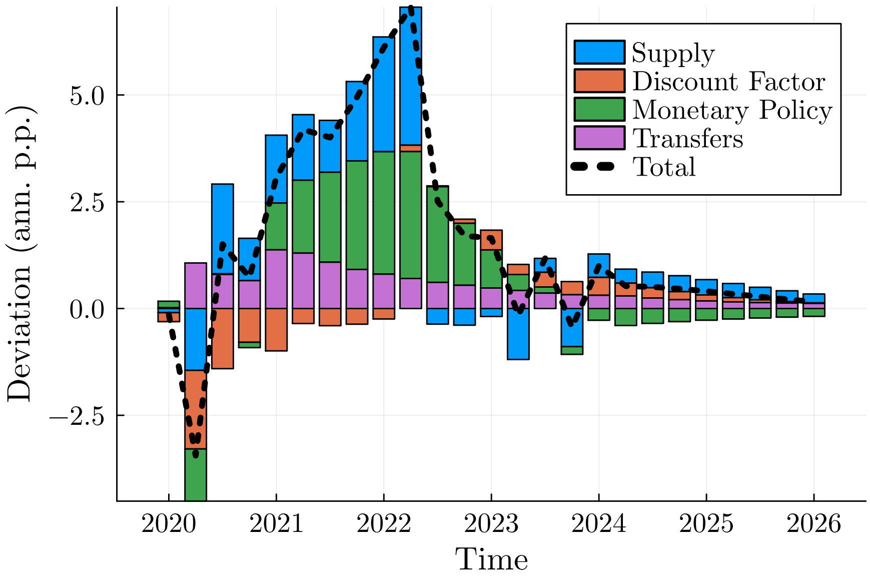
Note: B/Y represents the real market value of public debt B^g over 4 times (=annualized) GDP.

Figure E.11: Aggregates dynamics using filtered shocks: longer horizon



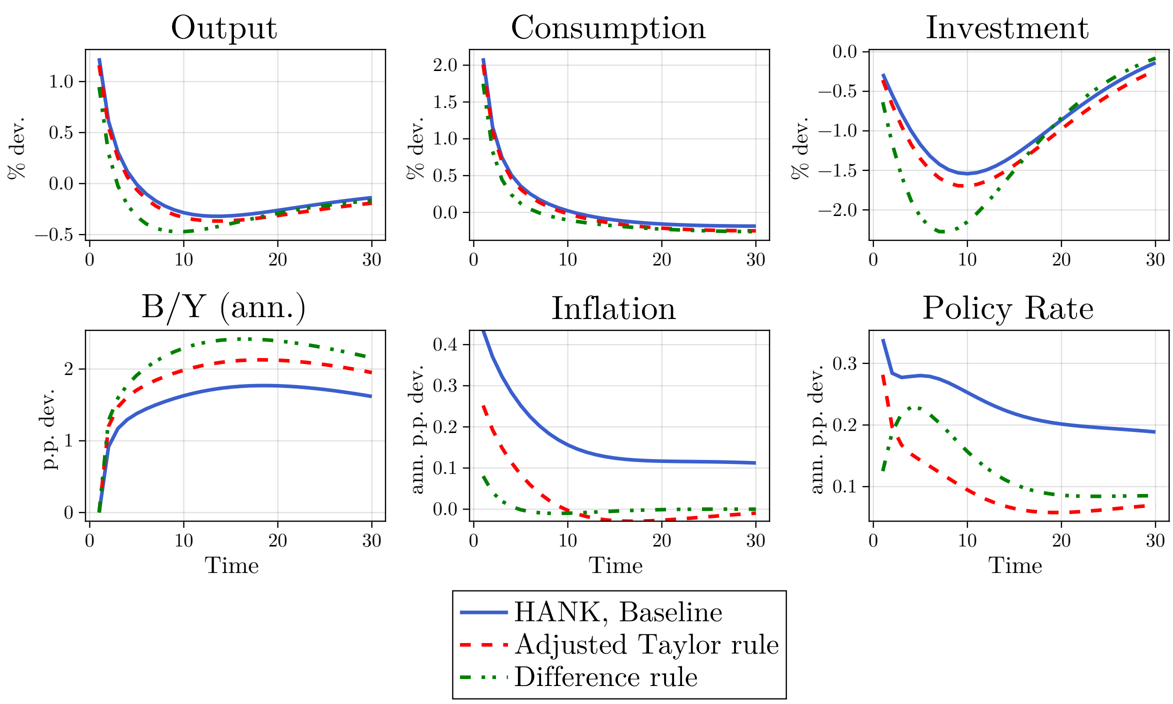
Note: B/Y represents the real market value of public debt B^g over 4 times (=annualized) GDP.

Figure E.12: Inflation decomposition: $\Psi = 0$ case



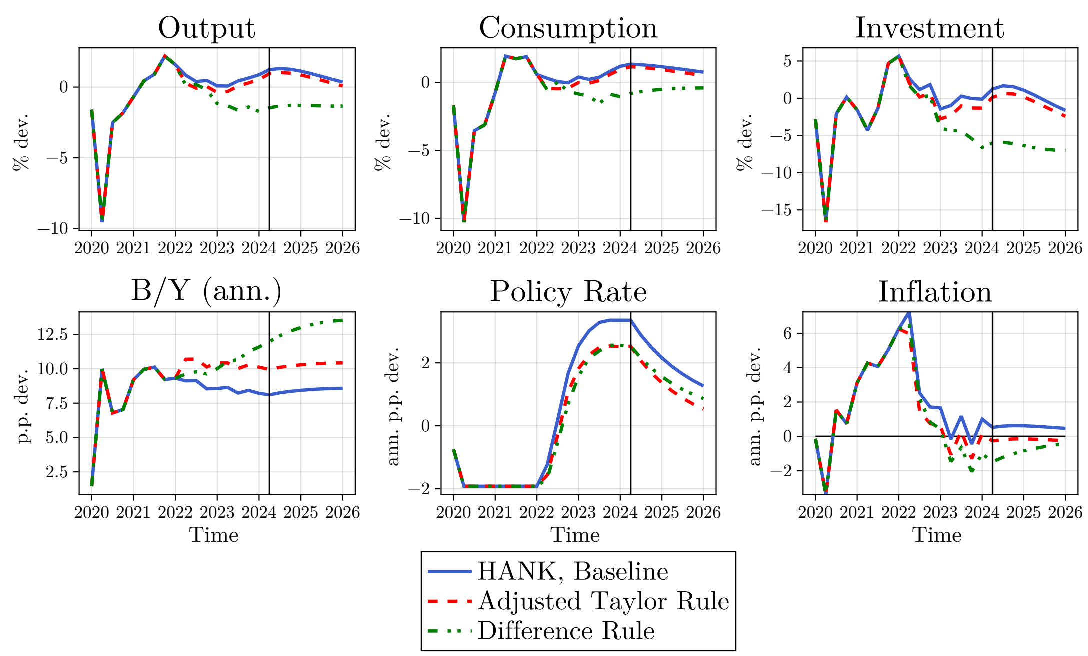
Note: B/Y represents the real market value of public debt B^g over 4 times (=annualized) GDP.

Figure E.13: Response to transfer shocks: Alternative rules



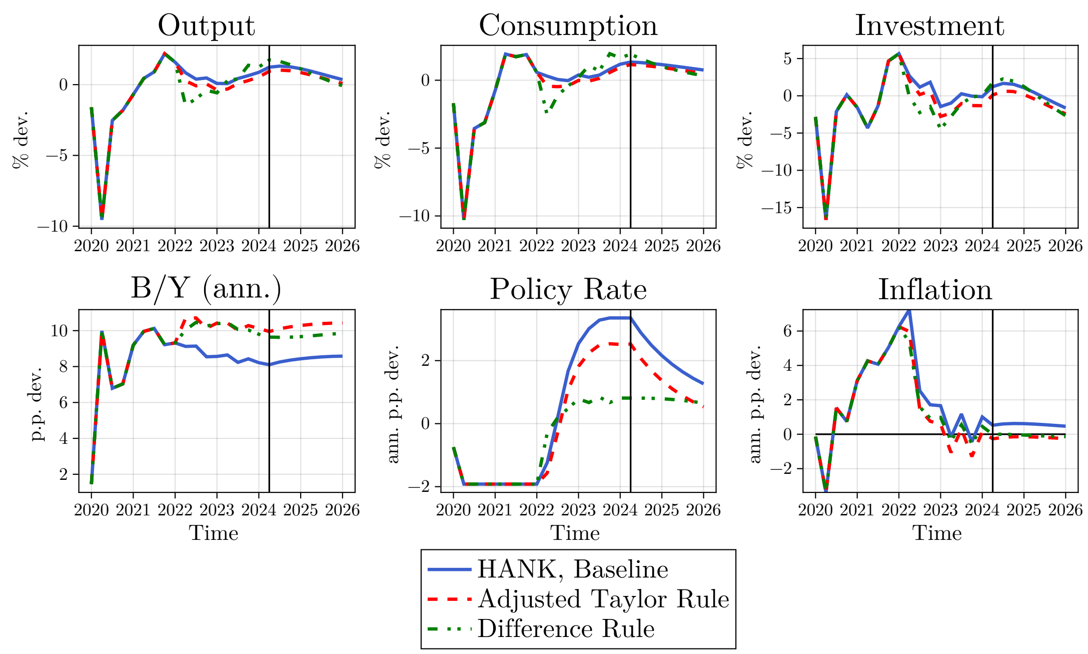
Note: B/Y represents the market value of public debt B^g over annualized GDP.

Figure E.14: Alternative rules: Difference rule with $\theta_\pi = 0.3$



Note: B/Y represents the real market value of public debt B^g over 4 times (=annualized) GDP.

Figure E.15: Alternative rules: Difference rule with $\theta_\pi = 0.3$ and no monetary shocks



Note: B/Y represents the real market value of public debt B^g over 4 times (=annualized) GDP.