Monetary Policy Transmission, Central Bank Digital Currency, and Bank Market Power*

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Abstract

Interest rates on new central bank digital currencies (CBDCs) can be expected to enter the monetary policy toolkit soon. Using an extended Sidrauski (1967) model featuring an oligopolistic banking sector, we study the complex transmission of CBDC rate adjustments, which generally involve both *direct* and *indirect* effects. This is because a CBDC rate cut does not only affect the rate on the CBDC itself, but also induces the non-competitive deposit providers to adjust their spreads, as the new substitute for their products becomes relatively less attractive. A calibration exercise suggests that the indirect effects depend strongly on deposit market concentration: If sufficiently high, they can provide substantial real effects even in a scenario with limited CBDC adoption. This contrasts them with traditional reserve rate policies which are weakened by a less competitive banking sector. Our framework also yields insights into optimal long-run monetary policy in the presence of CBDC and bank market power.

Keywords: CBDC, Digital Currency, Bank Market Power, Monetary Transmission

JEL Codes: E42, E43, E52

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1 Introduction

Monetary authorities around the world are exploring the possibility of issuing a new digital payment instrument widely accessible to the public. As of July 2021, 56 central banks have publicly communicated research or development efforts on central bank digital currency (CBDC) (Auer et al., 2022). Motivations for such a new payment instrument include ensuring adequate public money, reducing systemic risk and improving financial stability, increasing competition in payments, and promoting financial inclusion (Engert and Fung, 2017).

One of the less-discussed aspects of CBDC is its potential to facilitate a direct implementation of monetary policy (e.g., Auer et al., 2022; Bank for International Settlement, 2020). Interest on CBDC could become a new policy instrument, allowing policy-makers to directly impact households' decisions and sidestep financial intermediaries. However, banks will not idly stand by if the central bank makes it more attractive to hold an asset providing similar services as their deposits. In turn, the actual equilibrium impact of CBDC rates should depend both on households' liquidity preferences and the response of the financial sector. As a step towards clarifying the transmission of such policies, we study an extended Sidrauski (1967) model similar to the framework proposed by Niepelt (2023), which provides a parsimonious framework close to standard business cycle theory.

Crucially, we depart from Niepelt (2023) along two dimensions. Firstly, we assume CBDC and bank deposits to be imperfect substitutes, resulting in a well-defined household portfolio choice problem. The degree of substitutability between CBDC and deposits for transaction purposes is ultimately a design choice by policy-makers. We believe, though, that there are good reasons to expect that in practice, CBDC would not be (almost) perfectly substitutable with bank deposits. For example, bank deposits are typically bundled with other financial services such as credit lines (e.g. overdraft facilities, credit cards), while CBDC may be perceived as offering more privacy and security. Other features such as the interoperability between CBDC and deposits and the ability to conduct international transactions might also limit practical substitutability (Bacchetta and Perazzi, 2022). Nevertheless, our framework nests the common assumption of perfect substitutability between CBDC and deposits as a limit case.

Secondly, we allow for a banking sector in which bank market power is derived from market concentration and households' imperfect ability to substitute between banks. We assume a common deposit market in which a set of non-competitive banks compete by offering differentiated deposits, but do not restrict it to be either monopsonistic (as e.g. in Niepelt, 2023) or monopsonistically competitive (as e.g. in Bacchetta and Perazzi, 2022). Rather, such settings are again nested as limit cases, allowing us to vary the degree of deposit market concentration in order to demonstrate its importance for the transmission of CBDC rates.

We consider the interest rates on CBDC and reserves as the main monetary policy instruments. In our simple model, both can be shown to affect the real allocation through the *average cost of liquidity*, but the influence of the CBDC spread consists of both a *direct* and an *indirect* effect. Clearly, an increase in the CBDC spread (relative to a risk-free rate) directly increases the households' cost of liquidity, as it makes holding the digital currency less attractive. However, the rising spread also enables banks to widen the spreads on the deposits they offer, as the alternative source of liquidity becomes comparatively less attractive. This introduces the indirect effect, reminiscent of the deposit channel of monetary policy proposed by Drechsler et al. (2017).

While the quantitative magnitude of the direct effect depends simply on the amount of CBDC households will choose to hold given its design, the strength of the indirect effect is more nuanced, depending crucially on the level of concentration in the deposit market. Intuitively, if deposit market concentration is low, individual banks are small and cannot affect the amount of CBDC households will choose to hold. In turn, changes in the CBDC spread have little impact on the equilibrium deposit spread and the indirect effect is small. On the other hand, if the deposit market is highly concentrated, banks can practically compete with CBDC and adjust their spreads more, making the indirect effect relatively large. Interestingly, a simple calibration exercise suggests that even in a scenario in which bank deposits remain the predominant payment instrument, policy-makers may hence be able to substantially affect real allocations through the CBDC rate if the deposit market is sufficiently concentrated.

In contrast, the potency of reserve rates as a monetary policy instrument *decreases* under higher deposit market concentration. Under the assumption that larger reserve holdings make it cheaper for banks to provide deposits, a higher reserve spread makes doing so more costly, causing banks to increase their deposit spreads and households' cost of liquidity. This has a smaller effect if concentration is high, as the spreads charged by less competitive banks is relatively more dependent on their demand schedule, which is not directly affected by reserve rate policy.

Our framework also allows us to study optimal policy, which, in the long run, is similar to Niepelt (2023) and follows a Friedman-rule type logic. The CBDC and reserve rates should be set such that the households' opportunity cost of holding CBDC and banks' opportunity cost of holding reserves equal their respective societal cost. In addition, a deposit subsidy should be extended to the banks to correct for distortion caused by bank market power. We find that the higher the market concentration, the lower the interest on reserves the government should offer to the banks. Moreover, higher bank market power, either through a higher level of market concentration or due to a decrease in the substitutability between banks, implies that the government must offer a larger deposit subsidy to banks.

Our work relates to the growing and recent literature on CBDC, which has studied these potential new payment instruments from a variety of perspectives. For example, Agur et al. (2022) analyze the trade-offs associated with CBDC design given heterogeneous household preferences over payment instruments and network effects regarding their use. They conclude that central banks should indeed issue interest-bearing CBDCs and choose their rate so that other payment instruments remain in use. Similarly, Keister and Sanches (2023) highlight trade-offs associated with CBDC design choices. In particular, they argue that a CBDC with a deposit-like design would have positive effects by increasing payment- and exchange efficiency, but may also decrease investment by inducing higher funding costs for banks. Piazzesi and Schneider (2022), in turn, warn that CBDC crowding out bank deposits may decrease efficiency in financial intermediation due to a complementarity between offering both deposits and credit lines. Other work has studied the impact of CBDC adoption on financial stability with differing findings, i.e. that CBDC may either improve financial stability (Fernández-Villaverde et al., 2021) or encourage banking panics (Williamson, 2022).

Given that we study CBDC in a set-up with non-competitive banks, our work is particularly related to Andolfatto (2021) and Chiu et al. (2019), which study the impact of CBDC introduction in the presence of a non-competitive banking sector. Andolfatto (2021) focuses on the impact of CBDC introduction on bank lending and economic activity and finds that a CBDC may not impede either. In fact, non-competitive banks forced to increase their deposit rates will be subject to an additional inflow of deposits due to the more attractive rates, and in turn, convert this additional funding into lending. Chiu et al. (2019) obtain similar results in a different set-up allowing for differing degrees of bank market power. Hence, in contrast to our work, these papers focus on the effect of CBDC on bank lending and general economic activity.

Jiang and Zhu (2021) and Garratt et al. (2022) share our focus by studying monetary pass-through in settings with imperfectly competitive or heterogeneous banks, respectively. Jiang and Zhu (2021) study the pass-through of both reserve and CBDC rates in a framework similar to Chiu et al. (2019). In the presence of a non-competitive banking sector, the introduction of CBDC is shown to potentially weaken the reserve pass-through, as perfect substitutability forces banks to match the CBDC rate on the deposit market. CBDC can essentially "dictate" the economy. The CBDC rate, in turn, may have a particularly strong pass-through to deposit rates, while its effects on loan rates depend on the reserve rate in an ambiguous way. A major difference between their work and ours is that due to the assumption of perfect substitutability between bank deposits and CBDC, Jiang and Zhu (2021) rule out the presence of the indirect effects discussed above, as the CBDC rate will either determine the deposit rate completely or not affect it all. Garratt et al. (2022) consider a framework with differing bank types ("large" and "small") competing for deposits from workers having heterogeneous preferences over the non-monetary benefits (e.g. extensive branch networks) they offer. They find that the pass-through of the CBDC rate to the deposit rate is stronger if the CBDC rate is high compared to the reserve rate, which, however, hurts the "small" bank. In contrast to our work, their focus is on bank heterogeneity, from which we abstract. Also, in their setup, no one actually ends up holding CBDC (the digital currency can again perfectly substitute for bank deposits and is out-competed by banks), so their model cannot provide for indirect effects of the CBDC rate on households' liquidity costs either.

Furthermore, our research is related to several studies analyzing the interaction of bank market power and monetary policy transmission. In particular, Drechsler et al. (2017) propose a deposit channel of monetary policy. As interest rate increases raise the opportunity costs of holding cash, non-competitive banks are able to increase the deposit spread in response to tighter monetary policy, consequently reducing the overall amount of deposits. This, in turn, can affect both the liquidity premium and bank lending. Estimating a structural model of the banking sector, Wang et al. (2022) similarly find bank market power to have important effects on the transmission of rate changes to deposit rates. In addition to the Drechsler et al. (2017) mechanism, their model also explicitly considers an oligopolistic lending market, where banks additionally respond by adjusting their lending rate markups.

The rest of the paper is organized as follows. Section 2 describes the elements of the model economy and characterizes its equilibrium. Section 3 analyzes the transmission mechanisms of the interest rates on CBDC and reserve. We first qualitatively characterize the channels through which the CBDC and reserve rates affect allocation. Then, we demonstrate quantitatively the extent to which market concentration in the deposit market affects policy transmissions. Section 4 derives the policy rules that support the first-best allocation of the economy. Section 5 discusses the robustness tests. Section 6 summarizes the results and concludes.

2 Model

We study an extended Sidrauski (1967) model, similar to Niepelt (2023), in which both the government and banks provide liquidity services to the households. Households substitute imperfectly between government-issued CBDC and commercial bank deposit services. Banks fund themselves by borrowing deposits from the households and invest in capital and reserves which are used to "back up" deposit issuance. We follow Drechsler et al. (2017) and assume that banks are non-competitive in the deposit market. Banks have market power due to market concentration and imperfect substitutability between banks' deposit services. Neoclassical firms produce a common consumption good using capital and labor, and a consolidated government/central bank issues CBDC and reserves.

2.1 Households

We consider an economy consisting of many identical and infinitely-lived households, with measure normalized to one. The representative household values consumption, c_t, liquidity services, z_{t+1}, and leisure, x_t, represented by an utility function of the form

$$u(c_t, z_{t+1}, x_t) = \frac{\left((1-\upsilon)c_t^{1-\psi} + \upsilon z_{t+1}^{1-\psi}\right)^{\frac{1-\sigma}{1-\psi}}}{1-\sigma} x_t^{\upsilon},$$

where $\upsilon \in (0,1)$ is the relative weight of liquidity services in utility, $\psi \in (0,1)$ is the inverse intratemporal elasticity of substitution between consumption and liquidity, and $\sigma > 0$ is the inverse intertemporal elasticity of substitution between consumption-liquidity bundles across time. We assume that CBDC and deposits are imperfect substitutes. Liquidity services are derived from real holdings of CBDC, m_{t+1}, and deposits, n_{t+1}, according to a Constant Elasticity of Substitution (CES) aggregator

$$z_{t+1} = \left((1-\gamma)m_{t+1}^{1-\epsilon} + \gamma n_{t+1}^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}},$$

where $\gamma \in (0,1)$ is the relative liquidity weight of bank deposits, and $\epsilon \in (0,1)$ is the inverse elasticity of substitution between CBDC and deposits. The liquidity weight parameter, γ, captures how useful deposits are for the purpose of holding liquidity relative to the same quantity of CBDC. We follow Drechsler et al. (2017) in assuming that deposits are themselves a composite good issued by a set of N non-competitive banks. Each bank i has mass $1/N$ and produces deposits of a quantity n^i_{t+1}/N. The household values deposits at different banks such that

$$n_{t+1} = \left(\frac{1}{N} \sum_{i=1}^{N} \left(n_{t+1}^i \right)^{1-\eta} \right)^{\frac{1}{1-\eta}}, \tag{1}$$

where η denotes the inverse elasticity of substitution between banks. The representative household can be thought of as an aggregation of many individual households who may have diverse preferences for holding deposits at different banks. Therefore, the representative household substitutes deposits imperfectly across banks, which implies that $\eta < 1$.

The household's budget constraint is given by

$$c_t + k_{t+1}^h + m_{t+1} + \frac{1}{N} \sum_{i=1}^{N} n_{t+1}^i + \tau_t = w_t(1 - x_t) + \pi_t + k_t^h R_t^k + m_t R_t^m + \frac{1}{N} \sum_{i=1}^{N} n_t^i R_t^{n,i}, \tag{2}$$

where k_{t+1}^h is holdings of capital, τ_t is the lump-sum tax net of government transfer, w_t is the wage rate, π_t is the dividends from firms and banks, R_t^k is the return on capital, R_{t+1}^m is the real gross interest rate on CBDC, and $R_{t+1}^{n,i}$ is the real gross interest rate on deposits at bank i. We assume that the returns on CBDC and deposits are risk-free, i.e. R_{t+1}^m and $R_{t+1}^{n,i}$ are known at time t. The household, taking prices, profits and taxes as given, solves

$$\begin{aligned}
&\max_{\{c_t, x_t, k_{t+1}^h, m_{t+1}, n_{t+1}^i\}_{t=0}^{\infty}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, z_{t+1}, x_t) \\
&\text{s.t.} \quad c_t + k_{t+1}^h + m_{t+1} + \frac{1}{N} \sum_{i=1}^{N} n_{t+1}^i + \tau_t = w_t(1 - x_t) + \pi_t + k_t^h R_t^k + m_t R_t^m + \frac{1}{N} \sum_{i=1}^{N} n_t^i R_t^{n,i}, \\
&\qquad k_{t+1}^h, m_{t+1}, n_{t+1}^i \geq 0.
\end{aligned}$$

We now turn to the first-order optimality conditions of the household program. Detailed derivations are provided in the appendix A.1. First, the household optimally allocates resources between deposits at individual banks according to

$$n_{t+1}^i = n_{t+1} \left(\frac{\chi_{t+1}^{n,i}}{\chi_{t+1}^n} \right)^{-\frac{1}{\eta}}, \tag{3}$$

which closely resembles demand equations for differentiated consumption goods commonly derived in New-Keynesian models. The relative share of deposits at bank i, n_{t+1}^i / n_{t+1}, must relate negatively to its corresponding relative cost, $\chi_{t+1}^{n,i} / \chi_{t+1}^n$. Here, $\chi_{t+1}^{n,i}$ is the interest-rate differential between the risk-free rate, R_{t+1}^f, and deposit rate offered by bank i

$$\chi_{t+1}^{n,i} = 1 - \frac{R_{t+1}^{n,i}}{R_{t+1}^f},$$

which represents the opportunity cost of holding deposits at bank i and which we hereafter refer to as *deposit spread*. The risk-free rate is defined in the standard way as the inverse of the expected value of the household's stochastic discount factor, Λ_{t+1},

$$R_{t+1}^f = \frac{1}{\mathbb{E}_t[\Lambda_{t+1}]}. \tag{4}$$

We further define χ^n_{t+1} to represent the index that can be shown to capture the deposit spread associated with one unit of the aggregate deposit bundle n_{t+1} given demand schedule (3):

$$\chi^n_{t+1} = \left(\frac{1}{N} \sum_{i=1}^{N} \left(\chi^{n,i}_{t+1} \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} . \tag{5}$$

This quantity can be interpreted as an aggregate price of deposits.

Next, optimization requires the marginal rate of substitution between consumption and each of the liquid assets to equal to their respective cost. These conditions can be combined to derive an expression for the velocity of consumption

$$\frac{c_t}{z_{t+1}} = \left(\frac{1-\upsilon}{\upsilon} \chi^z_{t+1} \right)^{\frac{1}{\psi}} , \tag{6}$$

where χ^z_{t+1} is a weighted average of the spreads on deposits and CBDC

$$\chi^z_{t+1} = \frac{\chi^m_{t+1} \chi^n_{t+1}}{\left((1-\gamma)^{\frac{1}{\epsilon}} \left(\chi^n_{t+1} \right)^{\frac{1-\epsilon}{\epsilon}} + \gamma^{\frac{1}{\epsilon}} \left(\chi^m_{t+1} \right)^{\frac{1-\epsilon}{\epsilon}} \right)^{\frac{\epsilon}{1-\epsilon}}} . \tag{7}$$

The *CBDC spread* is defined similarly to the deposit spread, as the interest-rate differential between the risk-free rate and the CBDC rate,

$$\chi^m_{t+1} = 1 - \frac{R^m_{t+1}}{R^f_{t+1}} \tag{8}$$

and denotes the opportunity cost of holding CBDC. Thus, we interpret χ^z_{t+1} as being the household's *average cost of liquidity.* Note that consumption velocity is increasing in this cost. As liquidity becomes more expensive, the household would want to economize on its liquidity holdings and therefore velocity increases. In the limiting case where the relative utility weight of liquidity goes to zero, i.e. $\upsilon \to 0$, consumption velocity goes to infinity and the model economy converges to the standard "cashless limit" case. Moreover, the household's demand for CBDC and deposits can be expressed as

$$\frac{m_{t+1}}{z_{t+1}} = (1-\gamma)^{\frac{1}{\epsilon}} \left(\frac{\chi^m_{t+1}}{\chi^z_{t+1}} \right)^{-\frac{1}{\epsilon}} , \tag{9}$$

$$\frac{n_{t+1}}{z_{t+1}} = \gamma^{\frac{1}{\epsilon}} \left(\frac{\chi^n_{t+1}}{\chi^z_{t+1}} \right)^{-\frac{1}{\epsilon}} . \tag{10}$$

We see that the relative demand for each of the liquid assets is increasing in their relative liquidity weights and decreasing in their costs relative to the average cost. In the special case where the liquidity weight of deposits goes to zero or the relative cost of deposits goes to infinity, CBDC becomes the household's only source of liquidity, i.e. $z_{t+1} = m_{t+1}$. The opposite occurs if the weight of deposits goes to one or the relative cost of CBDC goes to infinity.

Lastly, we derive the Euler equation for capital and the leisure choice condition. The Euler equation

$$c_t^{-\sigma} x_t^{\upsilon} \Omega_t^c = \beta \mathbb{E}_t \left[R_{t+1}^k c_{t+1}^{-\sigma} x_{t+1}^{\upsilon} \Omega_{t+1}^c \right], \tag{11}$$

has the standard interpretation that equates the current-period marginal cost of savings, given by the marginal utility of consumption (left-hand side), with the next-period discounted expected return on savings (right-hand side). Notice that relative to the baseline real business cycle (RBC) model, the Euler equation (11) contains the term Ω_t^c

$$\Omega_t^c = (1-\varkappa)^{\frac{1-\sigma}{1-\psi}} \left(1 + \left(\frac{\varkappa}{1-\varkappa} \right)^{\frac{1}{\psi}} \left(\chi_{t+1}^z \right)^{1-\frac{1}{\psi}} \right)^{\frac{\psi-\sigma}{1-\psi}}, \tag{12}$$

which summarizes the impact of liquidity services on the marginal utility of consumption. Similarly, the leisure choice condition closely parallels its RBC counterpart

$$\frac{c_t^{1-\sigma}}{1-\sigma} \upsilon x_t^{\upsilon-1} \Omega_t^x = w_t c_t^{-\sigma} x_t^{\upsilon} \Omega_t^c, \tag{13}$$

and equates the marginal utility of leisure (left-hand side) with its marginal cost in utility terms (right-hand side). Liquidity services influence the marginal utility of leisure through the quantity

$$\Omega_t^x = (1-\varkappa)^{\frac{1-\sigma}{1-\psi}} \left(1 + \left(\frac{\varkappa}{1-\varkappa} \right)^{\frac{1}{\psi}} \left(\chi_{t+1}^z \right)^{1-\frac{1}{\psi}} \right)^{\frac{1-\sigma}{1-\psi}}. \tag{14}$$

2.2 Banks

There is a set of N non-competitive banks that produce differentiated deposit services, and invest in capital and reserves. The balance sheet of a typical bank is

$$k_{t+1}^i + r_{t+1}^i = n_{t+1}^i, \tag{15}$$

where k_{t+1}^i and r_{t+1}^i denote the bank's capital and reserve holdings, respectively. We follow Niepelt (2023) and assume that maturity transformation requires bank resources. Banks incur a cost per unit of deposit funding and a role for reserves is introduced by assuming that larger reserves holdings relative to deposits reduce these costs. Unlike in Niepelt (2023), we do not assume positive externalities of reserve holdings as it does not substantially affect our results. A bank's per-unit cost of deposit, ν_t^i, is thus a decreasing function of its reserves-to-deposits ratio, $\zeta_{t+1}^i = r_{t+1}^i / n_{t+1}^i$,

$$\nu_t^i \left(\zeta_{t+1}^i \right) = \omega + \phi \left(\zeta_{t+1}^i \right)^{1-\varphi},$$

where $\omega, \phi \geq 0$; $\varphi > 1$. We assume that all banks face the same cost function.

At time t, bank i decides on its reserve holdings and deposit rate, subject to its deposit demand schedule (3) and the balance sheet constraint (15). Returns on the bank's assets, net of interest payments, are realized in the subsequent period. The bank retains no earnings and distributes its entire profit to the household every period. The date-t program of a typical bank is

$$\begin{aligned}
\max_{r^i_{t+1}, R^{n,i}_{t+1}} \quad & -n^i_{t+1}\nu^i_t + \mathbb{E}_t\left[\Lambda_{t+1}\left(k^i_{t+1}R^k_{t+1} + r^i_{t+1}R^r_{t+1} - n^i_{t+1}R^{n,i}_{t+1}\right)\right] \\
\text{s.t.} \quad & n^i_{t+1} = n_{t+1}\left(\frac{\chi^{n,i}_{t+1}}{\chi^n_{t+1}}\right)^{-\frac{1}{\eta}}, \\
& k^i_{t+1} = n^i_{t+1} - r^i_{t+1},
\end{aligned}$$

where R^r_{t+1} is the gross interest rate on reserve balances.

We focus on a symmetric industry equilibrium. Since all banks are identical they will choose identical deposit rates (and thus identical deposit spreads) and levels of reserve holdings, i.e. $\chi^{n,i}_{t+1} = \chi^{n,j}_{t+1}$ and $r^i_{t+1} = r^j_{t+1}$ for all i and j. Identical deposit spread across banks, given banks' demand schedule (3), implies that the household's demand for each bank's deposits is also identical, i.e. $n^i_{t+1} = n^j_{t+1}$ for all i and j. Using equations (1) and (5), we can establish that $n_{t+1} = n^i_{t+1}$ and $\chi^n_{t+1} = \chi^{n,i}_{t+1}$. Moreover, identical levels of reserve holdings mean that the aggregate reserve holdings of the whole banking sector are $r_{t+1} = (1/N)\sum_{i=1}^N r^i_{t+1} = r^i_{t+1}$. Then, the reserves-to-deposits ratios have to be equal across banks too, i.e. $\zeta^i_{t+1} = \zeta_{t+1}$. As all banks are identical, we hereafter drop the individual superscript i.

We now turn to the first-order conditions of the bank's optimization problem. Detailed derivations are provided in the appendix A.2. Firstly, the first-order condition with respect to reserves yields the bank's desired reserves-to-deposits ratio

$$\zeta_{t+1} = \left(\frac{\chi^r_{t+1}}{\phi(\varphi-1)}\right)^{-\frac{1}{\varphi}}. \tag{16}$$

The ratio depends on the *reserve spread* which represents the opportunity cost of holding reserves

$$\chi^r_{t+1} = 1 - \frac{R^r_{t+1}}{R^f_{t+1}}. \tag{17}$$

As the reserve spread increases and reserves become more expensive, the bank's desired reserves-to-deposits ratio decreases and its cost of deposit issuance increases. Equation (16) also shows that the bank's choice of reserves equalizes their marginal (opportunity) cost of holding reserves, χ^r_{t+1}, to the marginal gain stemming from a lower cost of deposit issuance, $-(1-\varphi)\phi\zeta^{-\varphi}_{t+1}$.

Secondly, the first-order condition with respect to deposit rate yields the condition that determines the equilibrium deposit spread

$$\chi^n_{t+1} + \chi^n_{t+1}\left(-\frac{1}{N}\left(\frac{1-s_t}{\psi} + \frac{s_t}{\epsilon}\right) - \left(1-\frac{1}{N}\right)\frac{1}{\eta}\right)^{-1} = \omega + \varphi\phi\zeta^{1-\varphi}_{t+1}. \tag{18}$$

The right-hand side of equation (18) shows the marginal cost of deposit issuance. The marginal unit of deposit not only implies an extra cost of $\nu_t = \omega + \phi\zeta^{1-\varphi}_{t+1}$, but also increases the cost for inframarginal units, given by $-\frac{\partial \nu_t}{\partial \zeta_{t+1}}\zeta_{t+1}$. The two components add up to $\omega+\varphi\phi\zeta_{t+1}^{1-\varphi}$. The left-hand side of (18) shows the banks' marginal benefit of raising deposit funding. The first term on the left-hand side, χ_{t+1}^{n}, entails (if positive) a return in excess of the reference risk-free rate. That is, deposits are a cheap source of funding for the bank and the spread denotes a marginal gain from deposit issuance. However, recall that the deposit spread represents an (opportunity) cost for the household. The second term on the left-hand side shows the decrease in the spread the bank must make in order to incentivize the household to provide more deposits

$$\chi_{t+1}^{n}\left(-\frac{1}{N}\left(\frac{1-s_t}{\psi}+\frac{s_t}{\epsilon}\right)-\left(1-\frac{1}{N}\right)\frac{1}{\eta}\right)^{-1}<0. \tag{19}$$

Expression (19) can be thought of as a markup over marginal cost which the non-competitive bank imposes on the household. The deposit spread markup depends negatively on the elasticity of demand that the bank faces, given by

$$-\frac{1}{N}\left(\frac{1-s_t}{\psi}+\frac{s_t}{\epsilon}\right)-\left(1-\frac{1}{N}\right)\frac{1}{\eta}, \tag{20}$$

where $s_t \in [0,1]$ is a relative weight

$$s_t=(1-\gamma)^{\frac{1}{\epsilon}}\left(\frac{\chi_{t+1}^{z}}{\chi_{t+1}^{m}}\right)^{\frac{1-\epsilon}{\epsilon}}. \tag{21}$$

The elasticity of demand (20) shows that the changes in the demand for deposits are the sum of two effects. Firstly, suppose the bank decreases its deposit rate and thus widens its deposit spread. It raises the aggregate deposit spread index, χ_{t+1}^{n}, by the amount equal to its mass, $1/N$. This makes deposits more costly overall for the household and induces a substitution away from deposits at a rate $\left(-\frac{1-s_t}{\psi}-\frac{s_t}{\epsilon}\right)<0$. This aggregate effect is more pronounced in a more concentrated deposit market since the actions of each bank have a larger impact on the overall cost of deposits. Secondly, given a decrease in the deposit rate, its deposit spread increases by $1-1/N$ relative to the aggregate index. This induces an outflow of deposits from the bank at the rate of the elasticity of substitution between banks, $1/\eta$. This interbank effect is larger when the elasticity of substitution between banks is large or when market concentration is low.

Suppose deposits at different banks are perfectly substitutable, i.e. $\eta \to 0$. Then, the elasticity of demand goes to infinity. The bank becomes competitive and sets the deposit spread equal to its marginal cost of deposit issuance

$$\chi_{t+1}^{n}=\omega+\varphi\phi\zeta_{t+1}^{1-\varphi}. \tag{22}$$

Alternatively, suppose the deposit market is perfectly dispersed, i.e. $1/N \to 0$. The bank becomes monopsonistically competitive and charges a constant multiplicative markup $1/(1-\eta)$ over its marginal cost. Then, the deposit spread becomes

$$\chi_{t+1}^{n}=\frac{\omega+\varphi\phi\zeta_{t+1}^{1-\varphi}}{1-\eta}. \tag{23}$$

2.3 Firms

Competitive firms produce common consumption goods using capital and labor. The representative firm maximizes its profit by solving the following problem

$$\begin{aligned}\max_{k_t, l_t} \quad & a_t f(k_t, l_t) - k_t \left(R_t^k - 1 + \delta\right) - w_t l_t \\ \text{s.t.} \quad & f(k_t, l_t) = k_t^{\alpha} l_t^{1-\alpha},\end{aligned}$$

where α is the capital share of output, a_t is productivity, k_t and l_t are the firm's demand for capital and labor, and δ is the capital depreciation rate. The first-order conditions of the firm pin down the capital return and the wage rate, respectively,

$$R_t^k = 1 - \delta + a_t \alpha \left(\frac{k_t}{l_t}\right)^{\alpha - 1}, \tag{24}$$

$$w_t = a_t (1 - \alpha) \left(\frac{k_t}{l_t}\right)^{\alpha}. \tag{25}$$

2.4 Consolidated government

A consolidated government/central bank issues CBDC and reserves, and invests in capital. The government incurs a per-unit cost, μ, when issuing (and managing) CBDC and a per-unit cost, ρ, when issuing (and managing) reserves. The budget constraint of the government reads

$$k_{t+1}^g - m_{t+1}(1 - \mu) - \frac{1}{N} \sum_{i=1}^{N} r_{t+1}^i (1 - \rho) = k_t^g R_t^k + \tau_t - m_t R_t^m - \frac{1}{N} \sum_{i=1}^{N} r_t^i R_t^r, \tag{26}$$

where k_{t+1}^g is the government's capital holdings. We assume that the government sets the interest rates on CBDC and reserves, and the level of lump-sum tax. The specific way in which the government sets these interest rates will be discussed in detail in the next sections.

2.5 Market clearing and aggregate resource constraint

Market clearing in the labor market requires that firms' labor demand equals the household's labor supply

$$l_t = 1 - x_t.$$

Capital market clearing requires that the firms' capital demand equals the sum of the capital holdings of the household, banks and the government

$$k_t = k_t^h + \frac{1}{N} \sum_{i=1}^{N} k_t^i + k_t^g.$$

Lastly, total dividends distributed to the household must equal the sum of banks' and firms' profits

$$\pi_t = \frac{1}{N}\sum_{i=1}^{N}\left(-n_{t+1}^i\nu_t^i + k_t^i R_t^k + r_t^i R_t^r - n_t^i R_t^{n,i}\right) + a_t k_t^\alpha l_t^{1-\alpha} - k_t\left(R_t^k - 1 + \delta\right) - w_t l_t.$$

Aggregate resource constraint is derived by combining the budget constraints of the household and the government, market clearing conditions, and total dividends

$$c_t + k_{t+1} - k_t(1-\delta) = a_t k_t^\alpha l_t^{1-\alpha} - Q_t,$$

where

$$Q_t = m_{t+1}\mu + n_{t+1}\left(\nu_t + \zeta_{t+1}\rho\right).$$

The resource constraint has the standard interpretation that available output in the economy is split between consumption, c_t, and investment, $k_{t+1} - k_t(1-\delta)$. However, there are resource costs associated with the provision of liquidity to the household: μ per unit of CBDC and $\nu_t + \zeta_{t+1}\rho$ per unit of deposit. The resource cost of deposits has two terms because the banking sector incurs a cost of deposit issuance, ν_t, and the government incurs a cost of issuing reserves used by the banking sector, $\zeta_{t+1}\rho$, to "back up" deposit issuance. The term Q_t summarizes the cost of liquidity provision. Because of these costs, the resources available for consumption and investment are less than the output, $a_t k_t^\alpha l_t^{1-\alpha}$. Since the household demands liquidity services in proportion to consumption, we can combine the terms c_t and Q_t, and rewrite the resource constraint as

$$c_t\Omega_t^{rc} + k_{t+1} - k_t(1-\delta) = a_t k_t^\alpha l_t^{1-\alpha}, \tag{27}$$

where $\Omega_t^{rc} \geq 1$ is given by

$$\Omega_t^{rc} = 1 + \left(\frac{\upsilon}{1-\upsilon}\frac{1}{\chi_{t+1}^z}\right)^{\frac{1}{\psi}}\left(\left((1-\gamma)\frac{\chi_{t+1}^z}{\chi_{t+1}^m}\right)^{\frac{1}{\epsilon}}\mu_t + \left(\gamma\frac{\chi_{t+1}^z}{\chi_{t+1}^n}\right)^{\frac{1}{\epsilon}}\left(\omega + \phi\zeta_{t+1}^{1-\varphi} + \zeta_{t+1}\rho_t\right)\right). \tag{28}$$

2.6 Policy and equilibrium

The consolidated government sets the interest rates on CBDC and reserves and elastically supplies these assets to the household and banks to meet demand. A policy consists of $\{R_{t+1}^m, R_{t+1}^r, \tau_t\}_{t\geq 0}$ and an equilibrium conditional on policy consist of

- a set of positive prices, $\{w_t, R_{t+1}^k, R_{t+1}^f, \chi_{t+1}^m, \chi_{t+1}^n, \chi_{t+1}^z, \chi_{t+1}^r\}_{t\geq 0}$;
- a positive allocation, $\{c_t, x_t, k_{t+1}\}_{t\geq 0}$;
- and positive CBDC, deposits and reserves holdings, $\{m_{t+1}, n_{t+1}, z_{t+1}, r_{t+1}\}_{t\geq 0}$,

such that (4), (6), (7), (8), (9), (10), (11), (13), (16), (17), (18), (24), (25) and (27) are satisfied.

3 CBDC and reserve rates as monetary policy tools

In this section, we analyze the transmission mechanisms of the interest rates on CBDC and reserves. We will first analytically characterize the channels through which the CBDC and reserve rates affect the allocation. Then, we will conduct a calibration exercise to gauge the quantitative importance of market concentration in the deposit market for policy transmissions. We do so by computing impulse responses of the economy to shocks to the CBDC and reserve rates under two different levels of market concentration.

3.1 Real effects of monetary policy

The three key conditions that characterize the equilibrium allocation, the Euler equation (11), the leisure choice condition (13), and the resource constraint (27), all closely parallel the conditions of a baseline RBC model. The differences relative to a RBC model are the quantities Ω^c_{t+1}, Ω^x_{t+1} and Ω^{rc}_{t+1}.

Importantly, the direct impact of liquidity on the household's consumption/savings and labor decisions, captured by Ω^c_{t+1} and Ω^x_{t+1}, depends solely on the average cost of liquidity, χ^z_{t+1}. So we will mostly focus on the effects of policy on χ^z_{t+1} when studying transmission below. For this purpose, it is instructive to first lay down how the average cost of liquidity works through our model economy.

The Euler equation (11) shows that the household's consumption/savings choices depend on liquidity through the marginal utility of consumption, which changes with the average cost of liquidity according to

$$\frac{\partial u_{c,t}}{\partial \chi^z_{t+1}} = c_t^{-\sigma} x_t \frac{\partial \Omega^c_t}{\partial \chi^z_{t+1}}, \text{ where } \frac{\partial \Omega^c_t}{\partial \chi^z_{t+1}} \propto \frac{\sigma - \psi}{\psi}.$$

We see that the sign of the impact on the marginal utility of consumption depends on the relative magnitudes of ψ and σ. If the household's intratemporal elasticity of substitution between consumption and liquidity is smaller than the intertemporal elasticity of substitution, i.e. $\psi > \sigma$, an increase in the cost of liquidity leads to a decrease in the marginal utility of consumption. This is driven by the fact that an increase in the cost of liquidity, according to (6), reduces the household's demand for it. A decrease in the level of liquidity then decreases the marginal utility of consumption, and hence there is consumption-liquidity complementarity. On the other hand, when $\psi < \sigma$ an increase in the cost of liquidity leads to an increase in the marginal utility of consumption, and consumption and liquidity are substitutes. In the case where $\psi = \sigma$, the household's utility is separable in consumption and liquidity and the cost of liquidity has no direct impact on consumption/savings choices.

The leisure choice condition (13) shows that the average cost of liquidity also impacts the household's labor supply choices through the marginal utility of leisure. Its partial derivative, with respect to the cost of liquidity, is given by

$$\frac{\partial u_{x,t}}{\partial \chi^z_{t+1}} = \frac{c_t^{1-\sigma}}{1-\sigma} \upsilon x_t^{\upsilon-1} \frac{\partial \Omega^x_t}{\partial \chi^z_{t+1}}, \tag{29}$$

where

$$\frac{\partial \Omega_t^x}{\partial \chi_{t+1}^z} \propto \frac{\sigma - 1}{\psi}. \tag{30}$$

Here, the sign of the effect on the marginal utility of leisure depends on the household's intertemporal elasticity of substitution between consumption-liquidity bundles across time. If the intertemporal elasticity of substitution is larger than one, i.e. $\sigma < 1$, an increase in the cost of liquidity decreases the marginal utility of leisure. On the other hand, if $\sigma > 1$ an increase in the cost of liquidity increases the marginal utility of leisure.

Finally, the spreads on CBDC and deposits also show up in the aggregate resource constraint (27), through Ω_t^{rc}. This reflects the resource costs associated with liquidity provision, incurred by the government and the banking sector. For instance, a change in the CBDC spread has two effects on the resource costs of liquidity provision. Firstly, it changes the average cost of liquidity which leads to an aggregate effect

$$\frac{\partial(\frac{z_{t+1}}{c_t})}{\partial \chi_{t+1}^m} \left(\frac{m_{t+1}\mu + n_{t+1}(\nu_t + \zeta_{t+1})}{z_{t+1}} \right).$$

The aggregate effect is the change in the resource costs driven by the change in the overall quantity of liquidity services, keeping its current composition into CBDC and deposits fixed. Secondly, there is a compositional effect

$$\frac{z_{t+1}}{c_t} \left(\frac{\partial \frac{m_{t+1}}{z_{t+1}}}{\chi_{t+1}^m}\mu + \frac{\partial \frac{n_{t+1}}{z_{t+1}}}{\chi_{t+1}^m}(\nu_t + \zeta_{t+1}) \right),$$

which is due to a rebalancing of the household's portfolio of liquid assets. This secondary effect is driven by the changes in the relative opportunity costs of CBDC and deposits.

In the special case where the household does not value liquidity services, i.e. $\upsilon \to 0$, Ω_{t+1}^c, Ω_{t+1}^x and Ω_{t+1}^{rc} all converge to one. At this "cashless limit", the cost of liquidity has no impact on the household's consumption/savings and leisure choices since no liquid assets are held. Therefore, there are also no resource costs associated with liquidity provision. Then, the model collapses into a standard RBC model.

To conclude, we have seen that the household's consumption/savings and leisure decisions and the economy's capital accumulation all depend on the average cost of liquidity, which in turn is a function of the spreads on CBDC and deposits. Therefore, the government can affect allocation only insofar as it affects these spreads. While the government controls the spread on CBDC by setting the CBDC rate, the deposit spread is determined by the banking sector. But as we will see below, the government can influence its behavior through the interest rates on *both* reserves and CBDC.

3.2 Interest on CBDC

We start by analyzing the channels through which the household's average cost of liquidity can be influenced by the CBDC rate. Suppose the government lowers the CBDC rate so that the CBDC spread widens.[1] Taking the first-derivative of the average cost of liquidity, given by (7), with respect to the CBDC spread yields

$$\frac{\partial \chi^z_{t+1}}{\partial \chi^m_{t+1}} = \underbrace{(1-\gamma)^{\frac{1}{\epsilon}} \left(\frac{\chi^m_{t+1}}{\chi^z_{t+1}}\right)^{-\frac{1}{\epsilon}}}_{\text{direct effect}} + \underbrace{\gamma^{\frac{1}{\epsilon}} \left(\frac{\chi^n_{t+1}}{\chi^z_{t+1}}\right)^{-\frac{1}{\epsilon}} \frac{\partial \chi^n_{t+1}}{\partial \chi^m_{t+1}}}_{\text{indirect effect}}. \tag{31}$$

The last expression shows that the CBDC spread works through two channels. Firstly, it directly increases the cost of liquidity by the first term. The strength of this direct effect is increasing in the relative liquidity weight of CBDC, $1-\gamma$, and decreasing in how costly CBDC is relative to the average cost of liquidity, $\chi^m_{t+1}/\chi^z_{t+1}$. Comparing the direct effect with the household's demand for CBDC (9), we see that it is just the share of CBDC in the total stock of liquidity, m_{t+1}/z_{t+1}. Intuitively, the more important CBDC is as a source of liquidity for the household the larger the impact of its cost on liquidity's average cost.

Secondly, the CBDC spread affects the cost of liquidity through the deposit side, given by the second term. The strength of this indirect effect is increasing in the relative liquidity weight of deposits, γ, and decreasing in how costly deposits are relative to the average, $\chi^n_{t+1}/\chi^z_{t+1}$. Comparing the indirect effect with the household's demand for deposits (10), we see that it is equivalent to the product of the share of deposits in the total stock of liquid, n_{t+1}/z_{t+1}, and the change in the deposit spread caused by a change in the CBDC spread, $\partial \chi^n_{t+1}/\partial \chi^m_{t+1}$. Analogous to the direct effect, the more important deposits are as a source of liquidity the larger is this indirect effect. However, the sign and the magnitude of the second effect also depend on how the banking sector responds to an increasing CBDC spread, captured by $\partial \chi^n_{t+1}/\partial \chi^m_{t+1}$.

Optimality condition (18) shows that the CBDC spread can influence the deposit spread through the bank's marginal benefit of deposit issuance (left-hand side). Specifically, CBDC spread affects the elasticity of demand for deposits that the bank faces, given by (20). The demand elasticity depends on a weighted average of the household's elasticities of substitution to consumption, $1/\psi$, and CBDC, $1/\epsilon$. The CBDC spread determines this average through the relative weight s_t, given by (21). Taking the partial derivative of the demand elasticity (20) with respect to the CBDC spread, we get

$$\frac{1}{N}\left(\frac{\partial s_t}{\partial \chi^m_{t+1}}\right)\left(\frac{1}{\psi} - \frac{1}{\epsilon}\right), \tag{32}$$

where

$$\frac{\partial s_t}{\partial \chi^m_{t+1}} = -\frac{1-\epsilon}{\epsilon}\frac{s_t(1-s_t)}{\chi^m_{t+1}} < 0.$$

The partial derivative shows that the marginal impact of CBDC spread is non-zero only if $\psi \neq \epsilon$. Intuitively, banks in the aggregate face competition from CBDC and consumption for the household's resources. Therefore, any outflow from deposits depends on the household's elasticities of substitution to CBDC and consumption. The CBDC spread only influences the relative importance of these two sources of deposits outflow, indicated by s_t. If the household finds it as easy to substitute from deposits to consumption as it does to CBDC,

[1] For simplicity, we assume here that the reserve spread is constant.

i.e. $\psi = \epsilon$, then the two sources of competition for the banks are equally important and the CBDC spread does not influence the banks' deposit spread. In such a case, the equilibrium spread is set equal to the marginal cost of deposit issuance plus a constant markup, similar to the case where banks are monopsonistically competitive.

In general, it seems reasonable to expect that deposits will be more substitutable with CBDC than with consumption, i.e. $\psi > \epsilon$. Then, an increase in the CBDC spread makes the demand elasticity for deposits (20) less negative in value, and in turn, decreases the marginal benefit of deposit issuance. The intuition is that when its spread widens, CBDC becomes a comparatively expensive source of liquidity and a larger fraction of potential substitution out of deposits will go to consumption (indicated by a decrease in s_t and more weight being put on $1/\psi$). The elasticity of demand moves closer to $1/\psi$, which is smaller than $1/\epsilon$, and thus decreases in absolute value. Therefore, an increase in the CBDC spread makes the household's demand for deposits less elastic. For banks with market power, a less elastic demand means that in order to attract additional deposits from the household, the spread needs to be lowered by more than before. That is, the marginal benefit of deposit issuance decreases. Given a fixed marginal cost, this implies that the equilibrium deposit spread increases. In other words, an increase in the CBDC spread is akin to giving banks more market power. Banks take advantage of this and charge a higher spread on deposits in equilibrium.

As we alluded to previously, market conditions in the deposit market also play a central role. If deposits at different banks are perfect substitutes or the deposit market is perfectly dispersed, the equilibrium deposit spread is determined without the influence of the CBDC spread. If the household does not differentiate between banks, each individual bank's choice of how much deposits to issue does not matter for the equilibrium spread, which will equal the marginal cost of deposit issuance (22): the market is competitive. Similarly, if the deposit market is perfectly dispersed the impact of each individual bank's spread on the aggregate deposit spread goes to zero. The deposit market becomes monopsonistically competitive. The deposit spread is set with a markup over marginal cost, which solely depends on the substitutability between banks, given by (23). In both cases, the government cannot use the CBDC spread to influence the banking sector.

To sum up, when the government decreases the CBDC rate and widens the CBDC spread, it directly increases the household's average cost of liquidity and affects allocation. Moreover, a higher CBDC spread increases the spread on bank deposits, provided that banks have sufficient market power, which in raises the household's cost of liquidity further. The transmission of the CBDC rate through the banking sector is similar to the deposit channel of monetary policy proposed by Drechsler et al. (2017). The authors describe a situation in which the household holds cash issued by the government and deposits issued by banks with market power. Policy-makers can induce an increase in the deposit spread by increasing the household's opportunity cost of holding cash, captured by the nominal interest rate on risk-free bonds. In our model, instead, the alternative to bank deposits is CBDC. The government can similarly affect banks' deposit spread by changing the household's opportunity cost of holding this alternative, i.e. its spread.

3.3 Interest on reserves

The interest on reserves affects the household's average cost of liquidity only through its impact on the deposit spread. Suppose the government decreases the reserve rate so that the reserve spread increases.[2] Taking the first derivative of the average cost of liquidity with respect to the reserve spread, we get

$$\frac{\partial \chi^z_{t+1}}{\partial \chi^r_{t+1}} = \gamma^{\frac{1}{\epsilon}} \left(\frac{\chi^n_{t+1}}{\chi^z_{t+1}} \right)^{-\frac{1}{\epsilon}} \frac{\partial \chi^n_{t+1}}{\partial \chi^r_{t+1}}.$$

Notice that the marginal impact of the reserve spread is very similar to the indirect effect of the CBDC spread in (31). This is not surprising since both effects work through the banking sector. The impact of the reserve spread can be written as the product of the share of deposits in the total stock of liquid, n_{t+1}/z_{t+1}, and the change in the deposit spread caused by the change in the reserve spread, $\partial \chi^n_{t+1}/\partial \chi^r_{t+1}$. Again, the more important deposits are as a source of liquidity the larger is this effect. But, its sign and the magnitude also depend on how the banking sector responds to an increasing reserve spread, captured by $\partial \chi^n_{t+1}/\partial \chi^r_{t+1}$.

Optimality condition (18) shows that the reserve spread influences the deposit spread through the marginal cost of deposit issuance (right-hand side). The partial derivative of the marginal cost with respect to reserve spread is

$$\frac{\partial(\omega + \varphi \phi \zeta_{t+1}^{1-\varphi})}{\partial \chi^r_{t+1}} = (1-\varphi)\varphi\phi\zeta_{t+1}^{-\varphi} \frac{\partial \zeta_{t+1}}{\partial \chi^r_{t+1}}.$$

We see that the reserves spread works by changing the bank's optimal reserve-to-deposit ratio, ζ_{t+1}. According to (16), an increase in the reserve spread makes reserves more expensive to hold and thus decreases the equilibrium reserve-to-deposit ratio

$$\frac{\partial \zeta_{t+1}}{\partial \chi^r_{t+1}} = -\frac{\zeta_{t+1}}{\varphi \chi^r_{t+1}} < 0.$$

Since $\varphi > 1$, we see that the marginal cost deposit issuance is increasing in the reserve spread. Then, the banks' optimality condition (18) implies that an increase in the reserve spread raises the equilibrium deposit spread. Moreover, unlike with the CBDC spread, the impact of the reserve spread is naturally not restricted to cases where banks have market power. Even when the deposit market is competitive (equation (22)) or monopsonistically competitive (equation (23)), increasing marginal cost resulting from a higher reserve spread would raise the equilibrium deposit spread.

3.4 Calibration Exercise: Policy Shocks

When analyzing the impact of the CBDC rate numerically below, we assume it to follow a log AR(1) process

$$\log(R^m_{t+1}) = (1-\rho^m)\log(R^m) + \rho^m \log(R^m_t) + e^m_t,$$

[2] For simplicity, we assume here that the CBDC spread is constant.

where ρ^m is the persistence parameter, R^m is the steady state CBDC rate, and e_t^m is the exogenous shock. The exogenous shock is non-zero in the first period of the simulation and returns to zero afterwards. In order to properly isolate the effect, when analyzing the CBDC rate we assume that the reserve rate is set so that the reserve spread is constant at its steady state level. Then, the reserve rate is given by

$$R_{t+1}^r = R_{t+1}^f(\beta R^r),$$

where R^r is the steady state reserve rate.

Similarly, when analyzing the impact of the reserve rate, we assume it follows a log AR(1) process

$$\log(R_{t+1}^r) = (1 - \rho^r)\log(R^r) + \rho^r \log(R_t^r) + e_t^r,$$

where ρ^r is the persistence parameter and e_t^r is the exogenous shock. Again, the exogenous shock is non-zero in the first period and returns to zero afterwards. We assume that the CBDC rate is set such that the CBDC spread is constant at its steady state level. Then, the CBDC rate is given by

$$R_{t+1}^m = R_{t+1}^f(\beta R^m).$$

3.5 Calibration Exercise: Parameters

We calibrate the model to the U.S. economy, with each model period interpreted as a quarter. Table 1 summarizes the baseline calibration. We use variables without time subscripts to denote their steady state value.

3.5.1 Household

The household's discount factor $\beta = 1.03^{-1/4}$ is set to be consistent with a risk-free rate of 3% per year. We calibrate the utility weight of liquidity $\varkappa = 0.018$ to match a consumption velocity of 1.147, to be in line with U.S. data (Del Negro and Sims, 2015). We assume that the inverse intertemporal elasticity of substitution, σ, is 0.5, and that consumption and liquidity services are complements. Therefore, the inverse intratemporal elasticity of substitution between consumption and liquidity, ψ, is set higher than σ, to 0.6. We follow Bacchetta and Perazzi (2022) and assume a "medium" degree of substitutability between CBDC and deposits, setting the inverse elasticity of substitution between the two, ϵ, to 1/6. For simplicity, we assume that it is as easy to substitute between deposit services at different banks as is to substitute between CBDC and deposits. Hence, the inverse elasticity of substitution between banks, η, is set to 1/6. The relative liquidity benefit of deposits, γ, is calibrated to 0.74, which implies a steady state CBDC-to-deposits ratio of 1/10. In the absence of implemented CBDC, this is meant to capture a situation in which CBDC constitutes only a small portion of the household's portfolio. The leisure function coefficient, υ, is set to deliver a steady state labor supply of approximately 1/3.

Table 1: Baseline calibration

Parameter	Value	Source/motivation
Household		
β	$(1.03)^{-1/4}$	Annual $R^f = 3\%$
ϑ	0.018	$c/z = 1.147$ (Del Negro and Sims, 2015)
σ	0.5	Assumption
ψ	0.6	$\psi > \sigma$
ϵ	1/6	Bacchetta and Perazzi (2022)
η	1/6	$\eta = \epsilon$
γ	0.74	$m/n = 1/10$
υ	0.85	$l \approx 1/3$
Banks		
ω	0.008	Niepelt (2023)
φ	1.5	Niepelt (2023)
ϕ	0.0008	$\zeta = 0.1945$ (Niepelt, 2023)
N	3	Drechsler et al. (2017)
Firms		
α	1/3	Standard value
δ	0.025	Standard value
Government		
ρ	0.0001	Niepelt (2023)
μ	0.0001	$\mu = \rho$
R^r	1.0024	$\chi^r = 0.00497$ (Niepelt, 2023)
R^m	1.0	Assumption

3.5.2 Banks

We set the first two parameters in the banks' operating cost function, ω and φ, to 0.008 and 1.5, respectively, following Niepelt (2023). The third bank operating cost parameter, ϕ is calibrated to 0.0008 to match a steady state reserves-to-deposits ratio of 0.1945. Concentration in the deposit market, $1/N$ in the model, can be proxied by the Herfindahl index (HHI) in reality. In the baseline, we set $N = 3$, resulting in a HHI of 1/3 due to symmetry. This is close to the average county-level HHI of 0.35 estimated by Drechsler et al. (2017) for the U.S. over the period from 1994 to 2013.

3.5.3 Firms and government

The productive sector is standard. The capital share of output, α, is 1/3, and the rate of capital depreciation, δ, is 0.025. We follow Niepelt (2023) and set the government's cost of issuing reserves to 0.0001, and we assume that the cost of issuing CBDC is equal to that of reserves. The steady state interest on reserves, $R^r = 1.0024$, is set such that the spread on reserves is $\chi^r = 0.00497$. The interest on CBDC is assumed to be non-interest bearing and thus $R^m = 1.0$.

3.6 Impulse responses

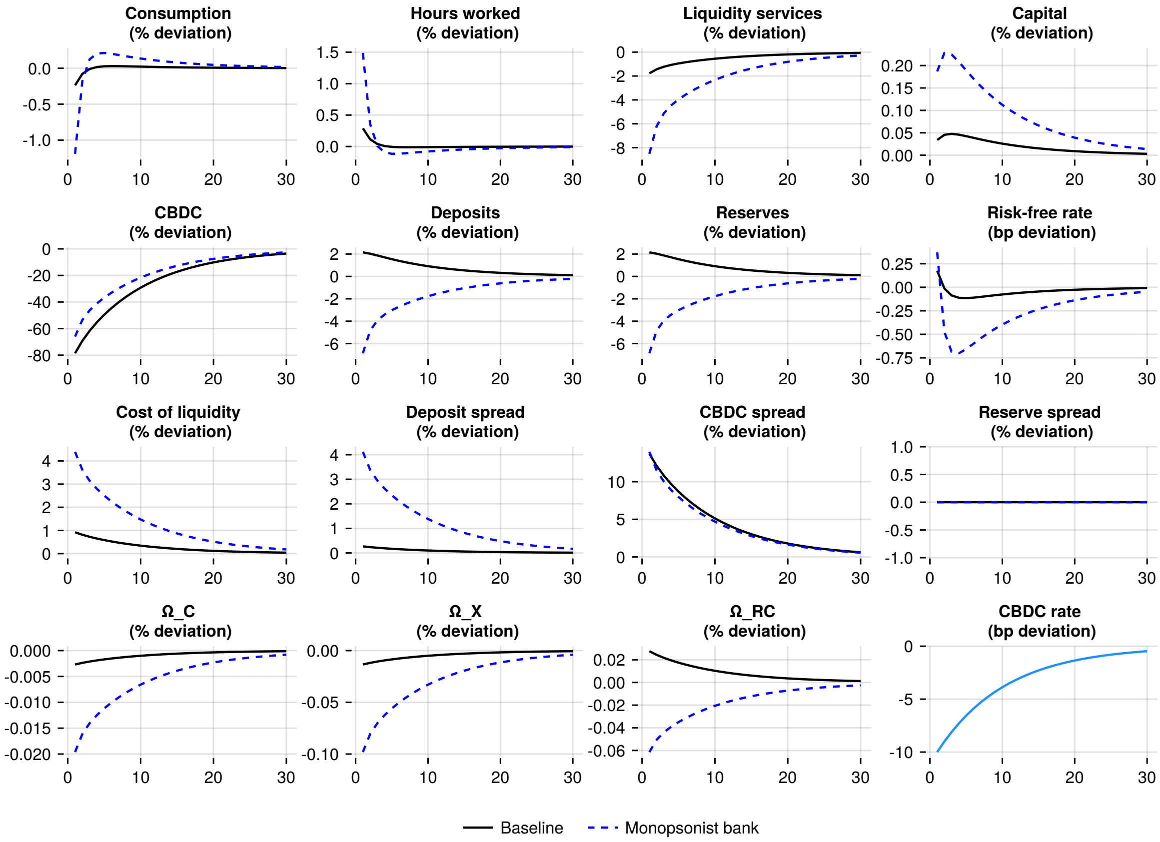
3.6.1 Response to a CBDC rate shock

Figure 1 shows the impulse responses, as deviations from the non-stochastic steady state, to a negative 10 basis points shock to the quarterly CBDC rate. First, a decrease in the CBDC rate immediately widens the CBDC spread by essentially the same magnitude as the risk-free rate changes little. The increasing CBDC spread raises the household's average cost of liquidity in both the baseline and the monopsonist bank case. This, in turn, decreases the household's demand for liquidity services as well as the household's current marginal utility of consumption, reflected in a lower Ω^c_{t+1}. In other words, the household's opportunity cost of saving, in utility terms, is reduced. The household is incentivized to save more and decrease current consumption. At the same time, decreasing the current marginal utility of consumption lowers the return from supplying labor in utility terms. This would, by itself, induce the household to choose more leisure. However, the marginal utility of leisure decreases more than the marginal utility of consumption: this is reflected in a larger initial decrease in Ω^x_{t+1} compared to Ω^c_{t+1}. The household's marginal benefit of leisure is now lower than its marginal cost and thus it increases labor supply. Lastly, with a higher cost of liquidity inducing the household to hold less of it, the resource costs being imposed on the economy's capital accumulation related to providing liquidity are also reduced. This contributes to the initial increase in capital holdings.

Importantly, there is a clear difference in the magnitudes of the responses with different levels of deposit market concentration. Here, we will focus on the cost of liquidity since it drives the changes in allocation. As we illustrated earlier, the impact of the CBDC spread on the cost of liquidity can be decomposed into a direct effect and an indirect effect, shown in (31). Figure 2 displays said decomposition of the responses of the cost of liquidity. The green dashed lines show the direct effects and the red solid lines show the indirect effects. The sum of the lines equals the original impulse responses of the cost of liquidity in figure 1.

In the baseline, where market concentration is relatively low, a 10 basis points decrease in the CBDC rate, and the subsequent increase in the CBDC spread, has a small impact on the average cost of liquidity and allocation. Both direct and indirect effects are small. We see from the left panel of figure 2 that while the indirect effect contributes non-negligibly to the initial response of the cost of liquidity, the direct effect remains dominant. Recall that the direct effect is equivalent to the ratio of CBDC to liquidity services, m_{t+1}/z_{t+1}. Since CBDC constitutes only a small fraction of the household's portfolio, the direct effect is small. Recall that the indirect effect is the product of the ratio of deposits to liquidity services, n_{t+1}/z_{t+1}, and the marginal change in the deposit spread induced by the CBDC spread, $\partial\chi^n_{t+1}/\partial\chi^m_{t+1}$. Interestingly, the indirect effect is also small despite the fact that the household's liquidity portfolio mainly consist of deposits. This is a result of the limited impact the CBDC spread has on the deposit spread as displayed in figure 1: When market concentration is low, banks can hardly react to the change in the CBDC spread due to competitive consid-

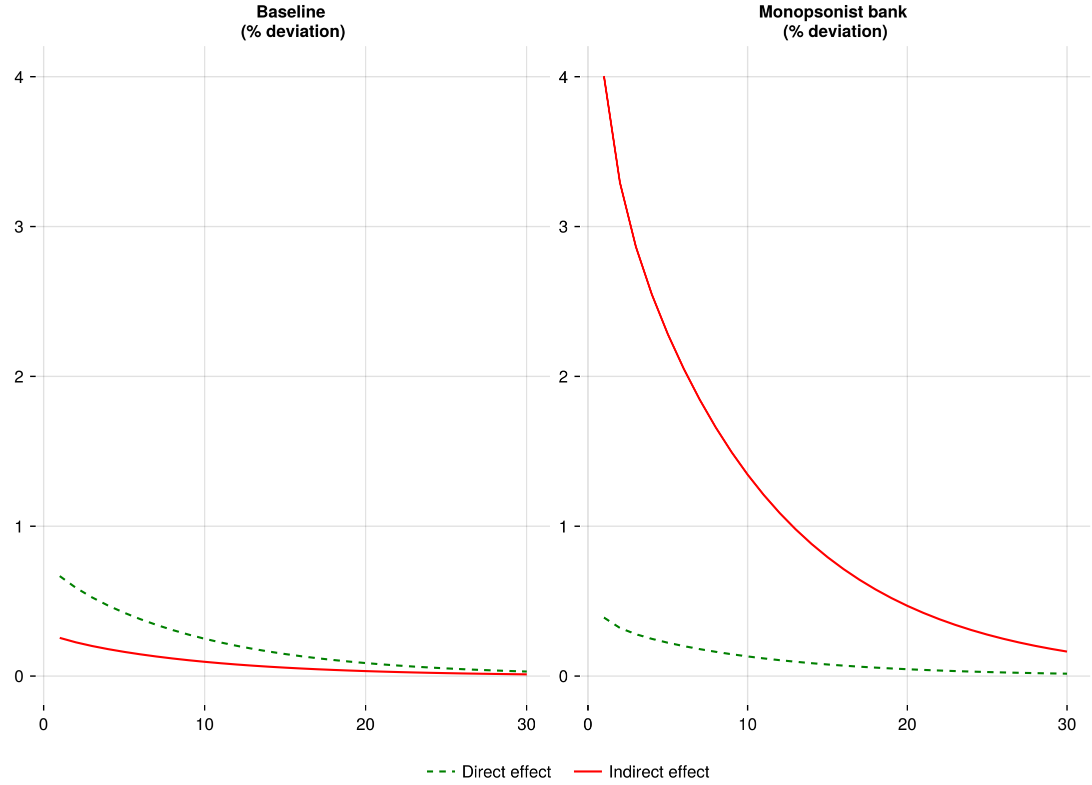
Figure 1: Impulse responses to 10 basis points decrease in CBDC rate



erations. In particular, the banks' demand elasticity for deposits (20) shows that they face external competition from CBDC and consumption, and interbank competition. The degree of external competition is captured by a weighted average of the household's elasticity of substitution to consumption, $1/\psi$, and CBDC, $1/\epsilon$. The CBDC spread only influences the relative importance of these two sources of deposit outflow. If market concentration is low, the "threat" from CBDC matters little for the banks. The equilibrium deposit spread is largely determined by the degree of interbank competition, captured by $1/\eta$. Intuitively, banks compete more with each other and less with the household's alternative source of liquidity, CBDC. Therefore, they are less responsive to the competitive pressure from CBDC and the equilibrium deposit spread is less sensitive to changes in the CBDC spread.

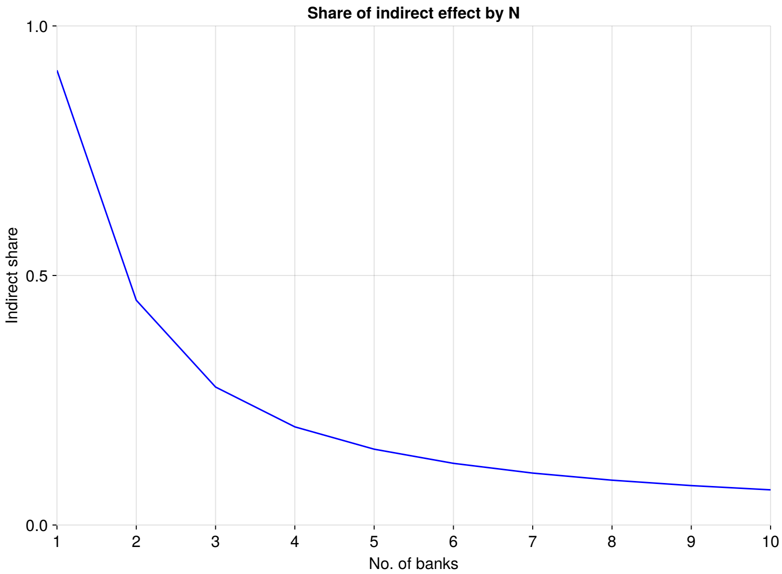
On the other hand, if the deposit market is highly concentrated, an increase in the CBDC spread generates a much larger increase in the cost of liquidity. The right panel of figure 2 shows that this is because the indirect effect is much larger than in the low concentration case. In general, high market concentration implies that the impact of each bank's action on the aggregate is larger. Then, banks compete less with each other and more with CBDC. In the extreme case where there is only one monopsonist bank, there is no interbank competition. Changes in the CBDC spread to a much larger extent enter into the banks' competitive considerations and the deposit spread is much more sensitive to the CBDC spread.

Figure 2: Decomposition of response of cost of liquidity



To further illustrate how deposit market concentration affects the model economy's response to the CBDC rate, figure 3 displays the indirect effect's relative contribution to the change of χ^z upon impact of the shock. As we already know from figure 2, the indirect effect dominates for the case of a banking monopoly ($N = 1$), but declines relatively quickly when allowing for more deposit providers. Nevertheless, even with a larger number of banks such as $N = 10$ the indirect effect's contribution remains firmly positive.

Figure 3: Relative sizes of the indirect effect by bank concentration

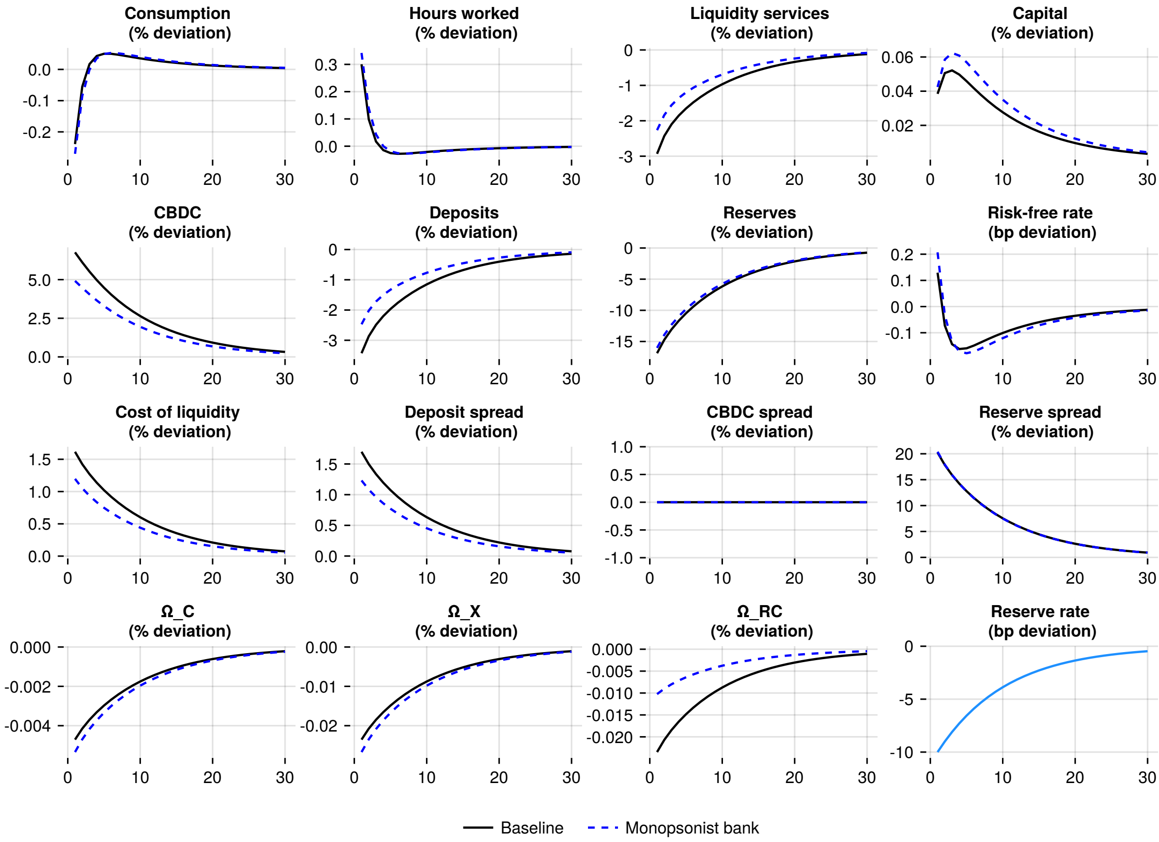


3.6.2 Response to a reserve rate shock

Figure 4 shows the impulse responses of the economy to a 10 basis points decrease in the reserve rate. The reserve spread widens as again, the risk-free rate barely moves. This immediately reduces the banks' demand for reserves as they become more expensive. The increasing reserve spread raises the cost of liquidity in both specifications. The higher cost of liquidity then affects allocation through the same mechanisms as described in the CBDC case above.

As we have shown earlier, the impact of the reserve spread on the cost of liquidity is the product of the ratio of deposits to liquidity services, n_{t+1}/z_{t+1}, and the marginal change in the deposit spread induced by the reserve spread, $\partial \chi^n_{t+1}/\partial \chi^r_{t+1}$. We see in figure 4 that the increasing reserve spread pushes up the deposit spread. This is because a higher reserve spread increases the banks' opportunity cost of holding reserves and thus reduces their optimal reserves-to-deposits ratio. In turn, it increases their marginal cost of deposit issuance and the banks pass on the increased cost to the household in the form of higher deposit spread. Notice that the increase in the deposit spread is larger in the baseline where market concentration is low. In that case, the banks' marginal benefit of deposit issuance (left-hand side of (18)) is less sensitive to changes in the deposit spread. For any increase in the deposit spread, the increase in the marginal benefit of deposit issuance is then lower. When an increase in the reserve spread increases the marginal operating cost (right-hand side of (18)), the banks would increase their deposit spread by more than in a case with higher market concentration. Intuitively, when banks have less market power, the "price" they charge on deposits is to a greater extent dictated by their marginal costs. The deposit spread is then more sensitive to changes in the reserve spread.

Figure 4: Impulse responses to 10 basis points decrease in reserve rate



4 Optimal policy rules

A Ramsey government can implement the first-best equilibrium allocation by setting policy instruments appropriately. In this section, we show the optimal policy rules and discuss the extent to which deposit market concentration impacts these rules.

The first-order conditions that characterize the first-best allocation can be found by solving the social planner problem, whereby the planner maximizes the household's utility subject to the aggregate resource constraint. The resulting social planner conditions are directly comparable to the first-order conditions of the household and banks. The consolidated government can implement the social planner solution by choosing policy instruments that support the optimal equilibrium. We restrict ourselves to a "first-order approach", in which policy instruments are set in such a way that the relevant first-order conditions in the competitive equilibrium are equivalent to the corresponding conditions in the social planner solution. Here, we focus on the main results and leave the details on the social planner problem and derivations of the rules to appendix A.6.

Firstly, the government should keep the CBDC spread, χ^m_{t+1}, equal to the government's per unit cost of issuing (and managing) CBDC, μ_t, at all times

$$\chi_{t+1}^{m*} = \mu_t.$$

Then, the CBDC rate should be set such that its spread, which is the household's opportunity cost of holding CBDC, is equal to the government's (societal) cost of issuing CBDC. Given the optimal target for the CBDC spread, we find the optimal rule for the CBDC rate

$$R_{t+1}^{m*} = R_{t+1}^{f}(1-\mu_t).$$

Next, the government should ensure that the reserve spread, χ_{t+1}^{r}, is equal to the government's per unit cost of issuing (and managing) reserves, ρ, adjusted for deposit market concentration

$$\chi_{t+1}^{r*} = \frac{1}{N}\rho_t. \tag{33}$$

The interpretation of the last expression is similar to that of the optimal CBDC spread. It is optimal for the government to keep each bank's opportunity cost of holding reserves, χ_{t+1}^{r}, equal to its share of the societal cost of providing reserves, ρ_t/N. The optimal rule for the reserve rate is then

$$R_{t+1}^{r*} = R_{t+1}^{f}\left(1-\frac{1}{N}\rho_t\right).$$

Lastly, the government should correct the distortion in the deposit market caused by bank market power. The efficient level of the deposit spread is that which would have prevailed if banks were competitive

$$\chi_{t+1}^{n*} = \omega + \varphi\phi_t\left(\zeta_{t+1}^{*}\right)^{1-\varphi},$$

where, given the optimal reserve spread, χ_{t+1}^{r*}, the banks' optimal reserves-to-deposits ratio is

$$\zeta_{t+1}^{*} = \left(\frac{1}{N}\frac{\rho_t}{\phi_t(\varphi-1)}\right)^{-\frac{1}{\varphi}}.$$

However, the government cannot control the deposit spread directly as it is determined by the banking sector. Specifically, it is determined by the bank optimality condition (18). The government can, nevertheless, offer banks a subsidy per unit of their deposit issuance, θ_t. The deposit subsidy should be set such that the last equation is fulfilled. Equating bank optimality condition (18) and the last expression for the optimal spread on deposits, we find the optimal level of subsidy

$$\theta_t^{*} = \chi_{t+1}^{n*}\left(\frac{1}{N}\left(\frac{1-s_t}{\psi}+\frac{s_t}{\epsilon}\right)+\left(1-\frac{1}{N}\right)\frac{1}{\eta}\right)^{-1},$$

which is the product of the optimal deposit spread and the inverse of the household's elasticity of demand for deposits (in absolute value).

The qualitative insights of the policy rules we derived are similar to those in Niepelt (2023). The spreads on CBDC and reserves should be targeted so that the opportunity costs of holding CBDC and reserves are equal to the societal costs of providing them. A deposit subsidy should be offered to the banks to eliminate distortion caused by bank market power. The key difference, however, is the importance of market concentration in our model. The reserve spread should be targeted in a way that depends on deposit market concentration, which is absent in Niepelt (2023). Higher market concentration means that the societal cost of providing reserves has to be divided between fewer banks. The burden on each bank is thus larger and their opportunity cost of reserves should reflect that. This means with a more concentrated deposit market the government should offer lower interest on reserves. Moreover, as we discussed before, with a more concentrated deposit market the household's demand for deposits is also less elastic. Then, banks have more market power and the optimal subsidy that corrects for that should also be larger. The same mechanism is also at work if the substitutability between banks is lower.

5 Robustness tests

A key uncertainty regarding the model is the relationship between CBDC and deposits. Since for most countries CBDC still largely remains a theoretical possibility, we are left to speculate regarding some aspects of the relationship. Therefore, we test the robustness of the results by changing (1) the substitutability between CBDC and deposits and (2) the steady state CBDC-to-deposit ratio. The main specifications assume a "medium" degree of substitutability between CBDC and deposits, i.e. $\epsilon = 1/6$, following Bacchetta and Perazzi (2022). The first test is then changing the degree of substitutability to $\epsilon = 1/5$ and $\epsilon = 1/7$.[3] Next, it is assumed in the main results that the steady state CBDC-to-deposits ratio is 1/10, to capture a scenario in which CBDC is a very small fraction of the household's portfolio. We test this assumption by changing this ratio to 1/5 and 1/15.

Figures 5 to 8 in the appendix show the impulse responses to a 10 basis points decrease in the CBDC rate with the alternative specifications described above. We see that the main takeaways from the previous section still stand. An increase in the CBDC spread reduces consumption, and increases labor supply and capital. Concentration in the deposit market still has an impact. Higher market concentration amplifies the impulse responses. Figures 9 to 12 in the appendix show the impulse responses to a 10 basis points decrease in the reserve rate. These figures show that while the main mechanisms through which a shock to the reserve rate affects allocation remain the same, the impact of deposit market concentration is somewhat less clear. In the lower substitutability and lower CBDC-to-deposits ratio cases, demonstrated by figures 9 and 11, the responses in the baseline are largely indistinguishable from those in high concentration case.

[3]In keeping with the main analysis, we set $\eta = \epsilon$

6 Conclusion

This paper studies the transmission mechanisms of monetary policy in the form of interest rates on CBDC and reserves in the presence of bank market power. In our framework, the CBDC rate affects allocation through its impact on the household's average cost of liquidity and its impact can be decomposed into a *direct* and an *indirect* effect. First, the CBDC spread directly affects the household's average cost of liquidity and in turn allocation. Second, an increase in the CBDC spread increases the average cost indirectly through its impact on the interest spread on deposits that banks charge. The size of the indirect effect depends on the level of market concentration in the deposit market. If market concentration is low, banks compete more with each other and less with CBDC. Changes in the CBDC spread have a small impact on the equilibrium deposit spread and the indirect effect is small. On the other hand, if the deposit market is highly concentrated, banks compete less with each other and more with CBDC. Thus, the equilibrium deposit spread responds more to changes in the CBDC spread and the indirect effect is large.

In contrast, the reserve rate affects the cost of liquidity, and thus allocation, only through its impact on the deposit spread that banks charge. An increase in the reserve spread raises banks' marginal cost of deposit issuance, and in turn they adjust equilibrium deposit rates. The increase in deposit spread caused by an increase in the reserve spread is larger when the deposit market is less concentrated. With limited market power, a larger fraction of the increase in marginal cost induced by a higher reserve spread is passed on to the household and results in a higher equilibrium deposit spread. Hence, our model suggests a key difference between CBDC- and reserve rates as monetary policy instruments: The effectiveness of the former is amplified by deposit market concentration, while the latter is diluted by it.

Finally, we also analyze the optimality of the model economy. We derive optimal policy rules that implement the first-best equilibrium allocation. The optimal policy rules are qualitatively similar to those found in Niepelt (2023). The CBDC and reserve rates should be set such that the household's opportunity cost of holding CBDC and banks' opportunity cost of holding reserves equal their respective societal cost. In addition, a deposit subsidy should be extended to the banks to correct for distortion caused by bank market power. However, our policy rules allow for conditions in the deposit market to play a role. First, we find that the higher the market concentration, the lower should the interest on reserves be. Second, higher bank market power, either through a higher level of market concentration or due to a decrease in the substitutability between banks, implies that the government must offer a larger deposit subsidy to banks.

Overall, our results suggest that the relative effectiveness of CBDC rates as a monetary policy instrument depends crucially on market power in the financial sector. In particular, a policymaker contemplating if or when to use them should keep deposit market concentration in mind. It thus also seems important for future research to incorporate oligopolistic banks into quantitative models featuring more realistic frictions and shocks.

A Derivations

A.1 Households

The household, taking prices, profits and taxes as given, solves

$$\max_{\{c_t, x_t, k^h_{t+1}, m_{t+1}, n^i_{t+1}\}_{t=0}^{\infty}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, z_{t+1}, x_t)$$

$$\text{s.t.} \quad c_t + k^h_{t+1} + m_{t+1} + \frac{1}{N}\sum_{i=1}^{N} n^i_{t+1} + \tau_t = w_t(1 - x_t) + \pi_t + k^h_t R^k_t + m_t R^m_t + \frac{1}{N}\sum_{i=1}^{N} n^i_t R^{n,i}_t,$$

$$k^h_{t+1}, m_{t+1}, n^i_{t+1} \geq 0.$$

Focusing on the interior solution, the first-order conditions with respect to capital, CBDC, deposits and leisure are

$$k^h_{t+1}: \quad 1 = \mathbb{E}_t\left[\Lambda_{t+1} R^k_{t+1}\right] \tag{A.1}$$

$$m_{t+1}: \quad \frac{u_{z,t} z_{m,t+1}}{u_{c,t}} = \chi^m_{t+1} \tag{A.2}$$

$$n^i_{t+1}: \quad \frac{u_{z,t} z_{n^i,t+1}}{u_{c,t}} = \frac{1}{N}\chi^{n,i}_{t+1} \tag{A.3}$$

$$x_t: \quad \frac{u_{x,t}}{u_{c,t}} = w_t, \tag{A.4}$$

where $f_{a,t}$ denotes the partial derivative of function f with respect to its argument a, Λ_{t+1} is the household's stochastic discount factor

$$\Lambda_{t+1} = \beta \frac{u_{c,t+1}}{u_{c,t}},$$

χ^m_{t+1} and $\chi^{n,i}_{t+1}$ are the CBDC spread and deposit spread at bank i, respectively,

$$\chi^m_{t+1} = 1 - \frac{R^m_{t+1}}{R^f_{t+1}}, \quad \chi^{n,i}_{t+1} = 1 - \frac{R^{n,i}_{t+1}}{R^f_{t+1}},$$

and the risk-free rate is defined as

$$R^f_{t+1} = \frac{1}{\mathbb{E}_t[\Lambda_{t+1}]}.$$

A.1.1 Demand for individual bank deposits

Household's first-order condition (A.3) with respect to deposits at any bank i can be written as

$$\frac{u_{z,t} z_{n,t+1}}{u_{c,t}} \left(\frac{n_{t+1}}{n^i_{t+1}}\right)^{\eta} = \chi^{n,i}_{t+1}. \tag{A.5}$$

Since the last expression holds for any bank, it means that for any two banks i and j

$$\chi_{t+1}^{n,i}\left(\frac{n_{t+1}^{i}}{n_{t+1}}\right)^{\eta}=\chi_{t+1}^{n,j}\left(\frac{n_{t+1}^{j}}{n_{t+1}}\right)^{\eta},$$

from which we find the demand for bank deposit j

$$n_{t+1}^{j}=\left(\frac{\chi_{t+1}^{n,i}}{\chi_{t+1}^{n,j}}\right)^{\frac{1}{\eta}}n_{t+1}^{i}. \tag{A.6}$$

Let T denote the sum of deposit spreads that the household incurs, and insert (A.6) into the expression,

$$T=\frac{1}{N}\sum_{i=1}^{N}n_{t+1}^{i}\chi_{t+1}^{n,i}=\frac{1}{N}\sum_{j=1}^{N}\left(\frac{\chi_{t+1}^{n,i}}{\chi_{t+1}^{n,j}}\right)^{\frac{1}{\eta}}n_{t+1}^{i}\chi_{t+1}^{n,j},$$

to find an expression for n_{t+1}^{i}

$$n_{t+1}^{i}=\frac{NT\left(\chi_{t+1}^{n,i}\right)^{-\frac{1}{\eta}}}{\sum_{j=1}^{N}\left(\chi_{t+1}^{n,j}\right)^{\frac{\eta-1}{\eta}}}. \tag{A.7}$$

We plug equation (A.7) into the definition of aggregate deposit, given by (1),

$$n_{t+1}=N^{\frac{\eta}{\eta-1}}T\left(\sum_{i=1}^{N}\left(\chi_{t+1}^{n,i}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{1-\eta}}. \tag{A.8}$$

Let χ_{t+1}^{n} be the spread associated with one unit of aggregate deposit, n_{t+1}. By setting $n_{t+1}=1$, we see from equation (A.8) that

$$\chi_{t+1}^{n}=\left(\frac{1}{N}\sum_{i=1}^{N}\left(\chi_{t+1}^{n,i}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}. \tag{A.9}$$

Given equation (A.9), we see that equation (A.7) can be written as

$$n_{t+1}^{i}=\frac{T}{\chi_{t+1}^{n}}\left(\frac{\chi_{t+1}^{n,i}}{\chi_{t+1}^{n}}\right)^{-\frac{1}{\eta}}, \tag{A.10}$$

and inserting the resulting expression into (1), we get

$$n_{t+1}=\left(\frac{1}{N}\sum_{i=1}^{N}\left(\frac{T}{\chi_{t+1}^{n}}\left(\frac{\chi_{t+1}^{n,i}}{\chi_{t+1}^{n}}\right)^{-\frac{1}{\eta}}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}=\frac{T}{\chi_{t+1}^{n}}. \tag{A.11}$$

Combining the expression for T and (A.11) we see that

$$T=\frac{1}{N}\sum_{i=1}^{N}n_{t+1}^{i}\chi_{t+1}^{n,i}=n_{t+1}\chi_{t+1}^{n}. \tag{A.12}$$

Lastly, inserting equation (A.12) into (A.10), we get the household's demand for deposits at bank i

$$n^i_{t+1} = n_{t+1} \left(\frac{\chi^{n,i}_{t+1}}{\chi^n_{t+1}} \right)^{-\frac{1}{\eta}}. \tag{A.13}$$

Combining the household's demand schedule with the first-order condition (A.5), we see that (A.5) can be expressed as

$$\frac{u_{z,t} z_{n,t+1}}{u_{c,t}} = \chi^{n,i}_{t+1} \left(\frac{n_{t+1}}{n^i_{t+1}} \right)^{-\eta} = \chi^n_{t+1}. \tag{A.14}$$

A.1.2 Optimality conditions

Given the functional form assumptions, the household's first-order conditions (A.2) and (A.14) become

$$m_{t+1}: \quad \frac{\upsilon z^{-\psi}_{t+1}}{(1-\upsilon)c^{-\psi}_t}(1-\gamma) \left(\frac{z_{t+1}}{m_{t+1}} \right)^{\epsilon} = \chi^m_{t+1} \tag{A.15}$$

$$n^i_{t+1}: \quad \frac{\upsilon z^{-\psi}_{t+1}}{(1-\upsilon)c^{-\psi}_t}\gamma \left(\frac{z_{t+1}}{n_{t+1}} \right)^{\epsilon} = \chi^n_{t+1}. \tag{A.16}$$

We combine (A.15) and (A.16) to get the ratio

$$\frac{m_{t+1}}{n_{t+1}} = \left(\frac{(1-\gamma)\chi^n_{t+1}}{\gamma \chi^m_{t+1}} \right)^{\frac{1}{\epsilon}}. \tag{A.17}$$

We plug equation (A.17) into CES function for z_{t+1} and solve for the ratio of z_{t+1} to n_{t+1}

$$\frac{z_{t+1}}{n_{t+1}} = \left(\frac{\left(\left((1-\gamma) \left(\chi^n_{t+1}\right)^{1-\epsilon} \right)^{\frac{1}{\epsilon}} + \left(\gamma \left(\chi^m_{t+1}\right)^{1-\epsilon} \right)^{\frac{1}{\epsilon}} \right)^{\frac{\epsilon}{1-\epsilon}}}{\gamma \chi^m_{t+1}} \right)^{\frac{1}{\epsilon}}. \tag{A.18}$$

Inserting (A.18) into equation (A.16) and solve for z_{t+1}, we get the household's optimal demand for liquidity

$$z_{t+1} = c_t \left(\frac{\upsilon}{1-\upsilon} \frac{1}{\chi^z_{t+1}} \right)^{\frac{1}{\psi}}, \tag{A.19}$$

where χ^z_{t+1} is the average cost of liquidity faced by the household

$$\chi^z_{t+1} = \frac{\chi^m_{t+1}\chi^n_{t+1}}{\left((1-\gamma)^{\frac{1}{\epsilon}} \left(\chi^n_{t+1}\right)^{\frac{1-\epsilon}{\epsilon}} + \gamma^{\frac{1}{\epsilon}} \left(\chi^m_{t+1}\right)^{\frac{1-\epsilon}{\epsilon}} \right)^{\frac{\epsilon}{1-\epsilon}}}.$$

Given household's optimal demand for z_{t+1}, we find the household's demand for m_{t+1} and n_{t+1}

$$
\begin{aligned}
m_{t+1} &= z_{t+1}\left((1-\gamma)\frac{\chi^z_{t+1}}{\chi^m_{t+1}}\right)^{\frac{1}{\epsilon}} \\
n_{t+1} &= z_{t+1}\left(\gamma\frac{\chi^z_{t+1}}{\chi^n_{t+1}}\right)^{\frac{1}{\epsilon}}.
\end{aligned} \tag{A.20}
$$

Plugging optimal z_{t+1}, given by (A.19), into the first-order conditions (A.1) and (A.4), we find the household's Euler equation and labor supply condition

$$
c_t^{-\sigma} x_t^{\upsilon} \Omega_t^c = \beta \mathbb{E}_t \left[R^k_{t+1} c_{t+1}^{-\sigma} x_{t+1}^{\upsilon} \Omega^c_{t+1} \right] \tag{A.21}
$$

$$
\frac{c_t^{1-\sigma}}{1-\sigma} \upsilon x_t^{\upsilon-1} \Omega_t^x = w_t c_t^{-\sigma} x_t^{\upsilon} \Omega_t^c, \tag{A.22}
$$

where Ω_t^c and Ω_t^x are given by

$$
\begin{aligned}
\Omega_t^c &= (1-\upsilon)^{\frac{1-\sigma}{1-\psi}} \left(1 + \left(\frac{\upsilon}{1-\upsilon}\right)^{\frac{1}{\psi}} \left(\chi^z_{t+1}\right)^{1-\frac{1}{\psi}}\right)^{\frac{\psi-\sigma}{1-\psi}} \\
\Omega_t^x &= (1-\upsilon)^{\frac{1-\sigma}{1-\psi}} \left(1 + \left(\frac{\upsilon}{1-\upsilon}\right)^{\frac{1}{\psi}} \left(\chi^z_{t+1}\right)^{1-\frac{1}{\psi}}\right)^{\frac{1-\sigma}{1-\psi}}.
\end{aligned}
$$

A.2 Banks

The date-t program of a typical bank is

$$
\begin{aligned}
\max_{r^i_{t+1}, R^{n,i}_{t+1}} \quad & -n^i_{t+1}\nu^i_t + \mathbb{E}_t\left[\Lambda_{t+1}\left(k^i_{t+1}R^k_{t+1} + r^i_{t+1}R^r_{t+1} - n^i_{t+1}R^{n,i}_{t+1}\right)\right] \\
\text{s.t.} \quad & n^i_{t+1} = n_{t+1}\left(\frac{\chi^{n,i}_{t+1}}{\chi^n_{t+1}}\right)^{-\frac{1}{\eta}} \\
& k^i_{t+1} = n^i_{t+1} - r^i_{t+1},
\end{aligned}
$$

where

$$
\nu^i_t\left(\zeta^i_{t+1}\right) = \omega + \phi\left(\zeta^i_{t+1}\right)^{1-\varphi}, \quad \zeta^i_{t+1} = \frac{r^i_{t+1}}{n^i_{t+1}}.
$$

The first-order conditions for bank i with respect to its deposit rate and reserve holdings are, respectively,

$$
R^{n,i}_{t+1}: \quad \chi^{n,i}_{t+1} + \frac{\chi^{n,i}_{t+1}}{e^{n,i}_{t+1}} = \nu^i_t - \nu^i_{\zeta,t}\zeta^i_{t+1} \tag{A.23}
$$

$$
r^i_{t+1}: \quad -\nu^i_{\zeta,t} = \chi^r_{t+1}, \tag{A.24}
$$

where $\chi^r_{t+1} = 1 - R^r_{t+1}/R^f_{t+1}$ and $e^{n,i}_{t+1}$ denotes the elasticity of demand for deposits at bank i with respect to its deposit spread, $\chi^{n,i}_{t+1}$, which in a symmetric industry equilibrium can be shown to be

$$e^{n,i}_{t+1} = \frac{\partial n^i_{t+1}}{\partial \chi^{n,i}_{t+1}} \frac{\chi^{n,i}_{t+1}}{n^i_{t+1}}.$$

Given functional form assumptions, the first-order condition (A.23) becomes

$$\chi^{n,i}_{t+1}\left(1 + \frac{1}{e^{n,i}_{t+1}}\right) = \omega + \varphi\phi\left(\zeta^i_{t+1}\right)^{1-\varphi},$$

where bank i's optimal reserves-to-deposits ratio is given by the first-order condition (A.24)

$$\zeta^i_{t+1} = \left(\frac{\chi^r_{t+1}}{\phi(\varphi - 1)}\right)^{-\frac{1}{\varphi}}.$$

To find the demand elasticity, $e^{n,i}_{t+1}$, we differentiate the household's demand for deposit at bank i, equation (A.13) with respect to $\chi^{n,i}_{t+1}$ and multiply it with the ratio $\chi^{n,i}_{t+1}/n^i_{t+1}$

$$\begin{aligned} e^{n,i}_{t+1} &= \left(-\frac{1}{\eta}\frac{n_{t+1}}{\chi^{n,i}_{t+1}}\left(\frac{\chi^n_{t+1}}{\chi^{n,i}_{t+1}}\right)^{\frac{1}{\eta}} + \frac{1}{\eta}\frac{n_{t+1}}{\chi^n_{t+1}}\left(\frac{\chi^n_{t+1}}{\chi^{n,i}_{t+1}}\right)^{\frac{1}{\eta}}\frac{\partial \chi^n_{t+1}}{\partial \chi^{n,i}_{t+1}} + \left(\frac{\chi^n_{t+1}}{\chi^{n,i}_{t+1}}\right)^{\frac{1}{\eta}}\frac{\partial n_{t+1}}{\partial \chi^n_{t+1}}\frac{\partial \chi^n_{t+1}}{\partial \chi^{n,i}_{t+1}}\right)\frac{\chi^{n,i}_{t+1}}{n^i_{t+1}} \\ &= -\frac{1}{\eta} + \frac{1}{\eta}\frac{\chi^{n,i}_{t+1}}{\chi^n_{t+1}}\frac{\partial \chi^n_{t+1}}{\partial \chi^{n,i}_{t+1}} + \left(\frac{\chi^{n,i}_{t+1}}{\chi^n_{t+1}}\right)^{\frac{1}{\eta}}\frac{\chi^{n,i}_{t+1}}{n^{n,i}_{t+1}}\frac{\partial n_{t+1}}{\partial \chi^n_{t+1}}\frac{\partial \chi^n_{t+1}}{\partial \chi^{n,i}_{t+1}} \end{aligned} \tag{A.25}$$

In a symmetric industry equilibrium, where $\chi^{n,i}_{t+1} = \chi^{n,j}_{t+1}$ and $n^i_{t+1} = n^j_{t+1}$ for any bank i and j,

$$\chi^n_{t+1} = \left(\frac{1}{N}\sum_{i=1}^{N}\left(\chi^{n,i}_{t+1}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} = \chi^{n,i}_{t+1}$$

$$n_{t+1} = \left(\frac{1}{N}\sum_{i=1}^{N}\left(n^i_{t+1}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}} = n^i_{t+1}$$

Then, equation (A.25) reduces to

$$e^{n,i}_{t+1} = \frac{1}{N}\left(\frac{\partial n_{t+1}}{\partial \chi^n_{t+1}}\frac{\chi^n_{t+1}}{n_{t+1}}\right) - \left(1 - \frac{1}{N}\right)\frac{1}{\eta}.$$

To find the aggregate demand elasticity, we differentiate household's optimal deposit demand, equation (A.20), with respect to the liquidity premium on deposits, χ^n_{t+1},

$$\frac{\partial n_{t+1}}{\partial \chi^n_{t+1}} = \frac{\partial z_{t+1}}{\partial \chi^z_{t+1}}\frac{\partial \chi^z_{t+1}}{\partial \chi^n_{t+1}}\left(\frac{\gamma\chi^z_{t+1}}{\chi^n_{t+1}}\right)^{\frac{1}{\epsilon}} + \frac{z_{t+1}\gamma}{\epsilon\chi^n_{t+1}}\frac{\partial \chi^z_{t+1}}{\partial \chi^n_{t+1}}\left(\frac{\gamma\chi^z_{t+1}}{\chi^n_{t+1}}\right)^{\frac{1-\epsilon}{\epsilon}} - \frac{z_{t+1}\gamma\chi^z_{t+1}}{\epsilon\left(\chi^n_{t+1}\right)^2}\left(\frac{\gamma\chi^z_{t+1}}{\chi^n_{t+1}}\right)^{\frac{1-\epsilon}{\epsilon}}$$

and multiply the last expression with the ratio χ^n_{t+1}/n_{t+1}

$$\frac{\partial n_{t+1}}{\partial \chi^n_{t+1}} \frac{\chi^n_{t+1}}{n_{t+1}} = -\frac{1}{\psi}\gamma^{\frac{1}{\epsilon}} \left(\frac{\chi^z_{t+1}}{\chi^n_{t+1}}\right)^{\frac{1-\epsilon}{\epsilon}} - \frac{1}{\epsilon}(1-\gamma)^{\frac{1}{\epsilon}} \left(\frac{\chi^z_{t+1}}{\chi^m_{t+1}}\right)^{\frac{1-\epsilon}{\epsilon}}.$$

Lastly, we write the optimality condition as it applies to a representative bank (and dropping the individual superscript i)

$$\chi^n_{t+1} + \chi^n_{t+1}\left(\frac{1}{N}\left(-\frac{1-s_t}{\psi} - \frac{s_t}{\epsilon}\right) - \left(1-\frac{1}{N}\right)\frac{1}{\eta}\right)^{-1} = \omega + \varphi\phi\zeta^{1-\varphi}_{t+1}, \tag{A.26}$$

where

$$\zeta_{t+1} = \left(\frac{\chi^r_{t+1}}{\phi(\varphi-1)}\right)^{-\frac{1}{\varphi}} \tag{A.27}$$

and $s_t \in [0,1]$ is

$$s_t = (1-\gamma)^{\frac{1}{\epsilon}} \left(\frac{\chi^z_{t+1}}{\chi^m_{t+1}}\right)^{\frac{1-\epsilon}{\epsilon}}.$$

A.3 Aggregate resource constraint

To find the aggregate resource constraint, we start by inserting total profit, π_t, into the household's budget constraint, imposing market clearing for labor and capital, and rearranging

$$\begin{aligned} k^h_{t+1} &= a_t k^\alpha_t (1-x_t)^{1-\alpha} + k_t(1-\delta) - c_t - m_{t+1} - \frac{1}{N}\sum_{i=1}^{N} n^i_{t+1} - \tau_t \\ &\quad - \frac{1}{N}\sum_{i=1}^{N} n^i_{t+1}\nu^i_t + \frac{1}{N}\sum_{i=1}^{N} r^i_t R^r_t - k^g_t R^k_t + m_t R^m_t. \end{aligned}$$

Next, from the government's budget constraint (26) we find an expression for k^g_{t+1}

$$k^g_{t+1} = m_{t+1}(1-\mu) + \frac{1}{N}\sum_{i=1}^{N} r^i_{t+1}(1-\rho) + k^g_t R^k_t + \tau_t - m_t R^m_t - \frac{1}{N}\sum_{i=1}^{N} r^i_t R^r_t.$$

We iterate forward capital market clearing condition

$$k_{t+1} = k^h_{t+1} + k^g_{t+1} + \frac{1}{N}\sum_{i=1}^{N}\left(n^i_{t+1} - r^i_{t+1}\right)$$

and plug in the expressions for k^h_{t+1} and k^g_{t+1} to get the aggregate resource constraint

$$k_{t+1} = a_t k^\alpha_t (1-x_t)^{1-\alpha} + k_t(1-\delta) - c_t - m_{t+1}\mu - \frac{1}{N}\sum_{i=1}^{N} n^i_{t+1}\nu^i_t - \frac{1}{N}\sum_{i=1}^{N} r^i_{t+1}\rho. \tag{A.28}$$

In a symmetric industry equilibrium, all banks choose the same balance sheet positions and $n_{t+1} = n^i_{t+1}$ and $r_{t+1} = r^i_{t+1}$, then the resource constraint becomes

$$k_{t+1} = a_t k_t^{\alpha}(1 - x_t)^{1-\alpha} + k_t(1 - \delta) - c_t - m_{t+1}\mu - n_{t+1}\nu_t - r_{t+1}\rho,$$

where

$$\nu_t = \omega + \phi\zeta_{t+1}^{1-\varphi}, \quad \zeta_{t+1} = \frac{r_{t+1}}{n_{t+1}}.$$

We can rewrite the resource constraint, using the definition of ζ_{t+1}, as

$$k_{t+1} = a_t k_t^{\alpha}(1 - x_t)^{1-\alpha} + k_t(1 - \delta) - c_t\Omega_t^{rc}, \tag{A.29}$$

where

$$\Omega_t^{rc} = 1 + \left(\frac{\upsilon}{1-\upsilon}\frac{1}{\chi^z_{t+1}}\right)^{\frac{1}{\psi}} \left(\frac{m_{t+1}}{z_{t+1}}\mu + \frac{n_{t+1}}{z_{t+1}}\left(\omega + \phi\zeta_{t+1}^{1-\varphi} + \zeta_{t+1}\rho\right)\right).$$

A.4 Steady state

Following the standard convention for the analysis of business cycle models, we analyze the effects of monetary policy by studying small policy perturbations around the economy's non-stochastic steady state, which we characterize here.

We denote steady state variables by dropping the time subscripts. In the steady state, the capital return and the risk-free rate are equal and given by the household's discount factor

$$R^k = R^f = \frac{1}{\beta}.$$

Conditional on policy, the CBDC and reserve spreads, χ^m and χ^r, are known. Then, the steady state deposit spread, χ^n, and reserves-to-deposits ratio, ζ, can be found using the bank optimality condition (A.26) and equation (A.27). Given the CBDC and deposit spreads, the cost of liquidity, χ^z and the quantities Ω^c, Ω^x and Ω^{rc} are also known. We divide the expression for capital return (24) by labor supply, $1-x$, to find the steady state capital-labor ratio in terms of primitives

$$\frac{k}{1-x} = \left(\frac{1}{a\alpha}\left(R^k - 1 + \delta\right)\right)^{\frac{1}{\alpha-1}}.$$

Notice that the steady state capital-labor ratio is identical to one that would have resulted in a baseline non-monetary RBC model. We divide resource constraint (A.29) by labor supply, $1-x$, to find the steady state consumption-labor ratio

$$\frac{c}{1-x} = \left(a\left(\frac{k}{1-x}\right)^{\alpha} - \delta\left(\frac{k}{1-x}\right)\right)\frac{1}{\Omega^{rc}}.$$

From the household's leisure choice condition (A.22), we can find the steady state consumption-leisure ratio

$$\frac{c}{x} = \frac{(1-\sigma)w}{\upsilon}\frac{\Omega_c}{\Omega_x},$$

where the steady state wage rate is also a function of the capital-labor ratio

$$w = a(1-\alpha)\left(\frac{k}{1-x}\right)^{\alpha}.$$

The household's preference for liquidity is reflected in the fact that the consumption-labor ratio and the consumption-leisure ratio are affected by the quantities Ω^c, Ω^x and Ω^{rc}, while the capital-labor ratio is not. The consumption-labor and consumption-leisure ratios can be combined to find the steady state leisure

$$x = \frac{c}{1-x}\left(\frac{c}{1-x} + \frac{c}{x}\right)^{-1}.$$

Given the steady state leisure, it is straightforward to back out the rest of the allocation and asset holdings; k, c, z, m, n and r.

A.5 Calibration

The parameters ϕ, φ, υ and μ are found in the following way. First, the banks' optimal reserves-to-deposits ratio is given by

$$\zeta = \left(\frac{\chi^r}{\phi(\varphi-1)}\right)^{-\frac{1}{\varphi}},$$

from which we find an expression for ϕ

$$\phi = \frac{\chi^r}{\zeta^{-\varphi}(\varphi-1)}. \tag{A.30}$$

The household's demand for CBDC and deposits implies that the deposit spread can be expressed as a function of the CBDC spread

$$\chi^n(\chi^m) = \frac{\gamma}{1-\gamma}\left(\frac{m}{n}\right)^{\epsilon}\chi^m.$$

Then, the cost of liquidity, $\chi^z(\chi^m, \chi^n) = \chi^z(\chi^m)$ is a function of the CBDC spread. Consequently, the left-hand side of the bank's optimality condition, denoted by LHS, is also a function of χ^m, which is known. The bank's optimality condition is

$$LHS = \varphi\phi\zeta^{1-\varphi}.$$

Plugging in equation (A.30) into the optimality condition

$$LHS = \varphi\frac{\chi^r}{\zeta^{-\varphi}(\varphi-1)}\zeta^{1-\varphi},$$

and rearranging, we find φ

$$\varphi = \frac{LHS}{LHS - \chi^r \zeta}.$$

Given φ, we can use (A.30) to find ϕ.

Next, the household's demand for liquidity services is given by

$$z = c\left(\frac{\upsilon}{1-\upsilon}\frac{1}{\chi^z}\right)^{\frac{1}{\psi}}.$$

Knowing the cost of liquidity, χ^z, and the desired inverse velocity, z/c, we find υ

$$\upsilon = \frac{\left(\frac{z}{c}\right)^{\psi}\chi^z}{1+\left(\frac{z}{c}\right)^{\psi}\chi^z}.$$

Lastly, we set μ equal to the total resource cost of supplying deposits

$$\mu = \omega + \phi\zeta^{1-\varphi} + \zeta\rho.$$

A.6 Optimality

The social planner maximizes the household's utility subject to the aggregate resource constraint (A.28)

$$\max_{\{c_t, x_t, k_{t+1}, m_{t+1}, n^i_{t+1}, r^i_{t+1}\}_{t=0}^{\infty}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, z_{t+1}, x_t)$$
$$\text{s.t.} \quad k_{t+1} = a_t k_t^{\alpha}(1-x_t)^{1-\alpha} + k_t(1-\delta) - c_t - m_{t+1}\mu - \frac{1}{N}\sum_{i=1}^{N} n^i_{t+1}\nu^i_t - \frac{1}{N}\sum_{i=1}^{N} r^i_{t+1}\rho.$$

The relevant first-order conditions are

$$k_{t+1}: \quad 1 = \mathbb{E}_t\left[\Lambda_{t+1}\left(a_{t+1} f_k(k_{t+1}, 1-x_{t+1}) + 1 - \delta\right)\right]$$

$$m_{t+1}: \quad \frac{u_{z,t} z_{m,t+1}}{u_{c,t}} = \mu_t \tag{A.31}$$

$$n^i_{t+1}: \quad \frac{u_{z,t} z_{n^i,t+1}}{u_{c,t}} = \frac{1}{N}(\nu^i_t - \nu^i_{\zeta,t}\zeta^i_{t+1}) \tag{A.32}$$

$$x_t: \quad \frac{u_{x,t}}{u_{c,t}} = a_t f_l(k_t, 1-x_t)$$

$$r^i_{t+1}: \quad -\nu^i_{\zeta,t} = \frac{1}{N}\rho_t. \tag{A.33}$$

Note that the social planner conditions are directly comparable to the first-order conditions of the household and banks.

We start by comparing the first-order condition with respect to CBDC in the household problem (A.2) and its social planner counterpart (A.31),

$$\begin{aligned} \text{Household}: \quad & \frac{u_{z,t} z_{m,t+1}}{u_{c,t}} = \chi^m_{t+1} \\ \text{Social planner}: \quad & \frac{u_{z,t} z_{m,t+1}}{u_{c,t}} = \mu_t. \end{aligned}$$

In order to replicate the social planner condition, the government should keep the spread on CBDC, χ^m_{t+1}, equal to the government's per unit cost of issuing (and managing) CBDC, μ_t, at all times

$$\chi^{m*}_{t+1} = \mu_t.$$

Next, we compare the first-order condition with respect to the reserve holdings of any bank i, r^i_{t+1}, in the bank problem (A.24) and its social planner counterpart (A.33),

$$\begin{aligned} \text{Bank}: \quad & -\nu^i_{\zeta,t} = \chi^r_{t+1} \\ \text{Social planner}: \quad & -\nu^i_{\zeta,t} = \frac{1}{N}\rho_t. \end{aligned}$$

To replicate the social planner condition, the government should ensure that

$$\chi^{r*}_{t+1} = \frac{1}{N}\rho_t.$$

Lastly, consider the first-order condition with respect to the deposit of bank i, n^i_{t+1}, in the household problem (A.3) and its social planner counterpart (A.32),

$$\begin{aligned} \text{Household}: \quad & \frac{u_{z,t} z_{n^i,t+1}}{u_{c,t}} = \frac{1}{N}\chi^{n,i}_{t+1} \\ \text{Social planner}: \quad & \frac{u_{z,t} z_{n^i,t+1}}{u_{c,t}} = \frac{1}{N}(\nu^i_t - \nu^i_{\zeta,t}\zeta^i_{t+1}), \end{aligned}$$

which we see are equalized if

$$\chi^{n^i*}_{t+1} = \nu^i_t - \nu^i_{\zeta,t}\zeta^i_{t+1}.$$

We recast the expression as it applies to a representative bank and get the expression for the optimal deposit spread

$$\chi^{n*}_{t+1} = \omega + \varphi\phi_t \left(\zeta^*_{t+1}\right)^{1-\varphi}, \tag{A.34}$$

where, given the optimal reserve spread, the banks' optimal reserves-to-deposits ratio is

$$\zeta^*_{t+1} = \left(\frac{1}{N}\frac{\rho_t}{\phi_t(\varphi-1)}\right)^{-\frac{1}{\varphi}}.$$

The government can offer a bank subsidy, θ_t, such that equation (A.34) is fulfilled. Equating bank condition (A.26) and optimal spread on deposits (A.34), we find the optimal level of subsidy

$$\theta^*_t = \chi^{n*}_{t+1}\left(\frac{1}{N}\left(\frac{1-s_t}{\psi} + \frac{s_t}{\epsilon}\right) + \left(1 - \frac{1}{N}\right)\frac{1}{\eta}\right)^{-1}.$$

B Additional figures

B.1 Robustness checks

Figure 5: Lower CBDC-deposits substitutability $\epsilon = \frac{1}{5}$

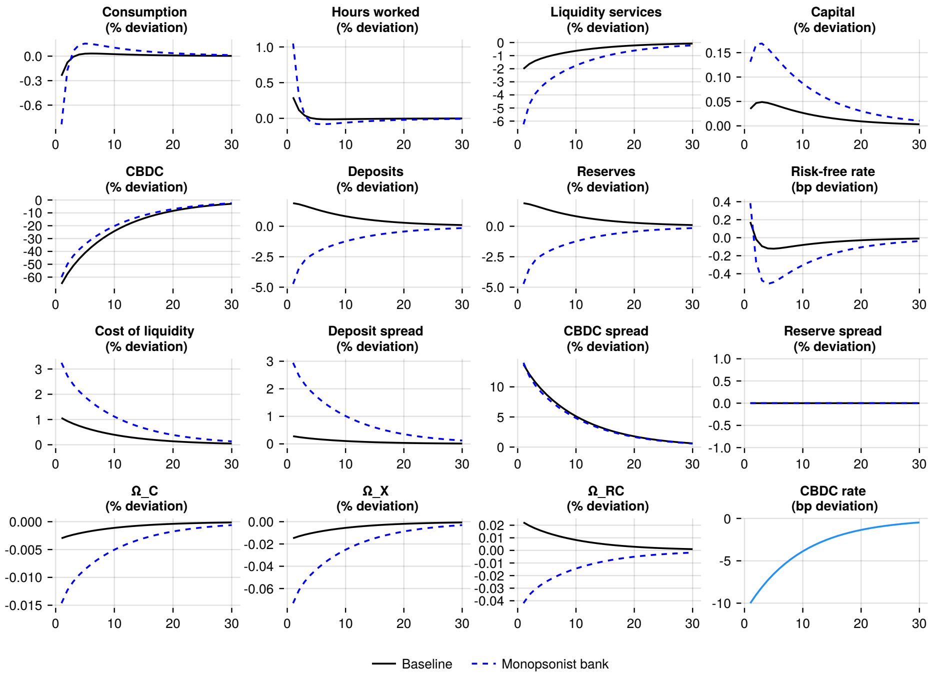


Figure 6: Higher CBDC-deposits substitutability $\epsilon = \frac{1}{7}$

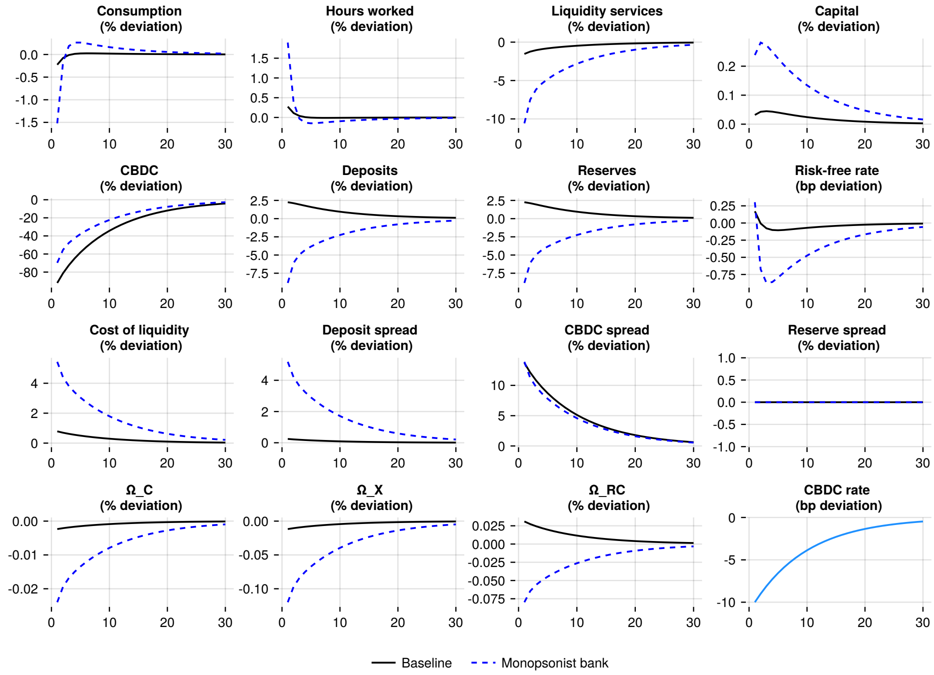


Figure 7: Lower steady state CBDC-deposit ratio $= \frac{1}{15}$

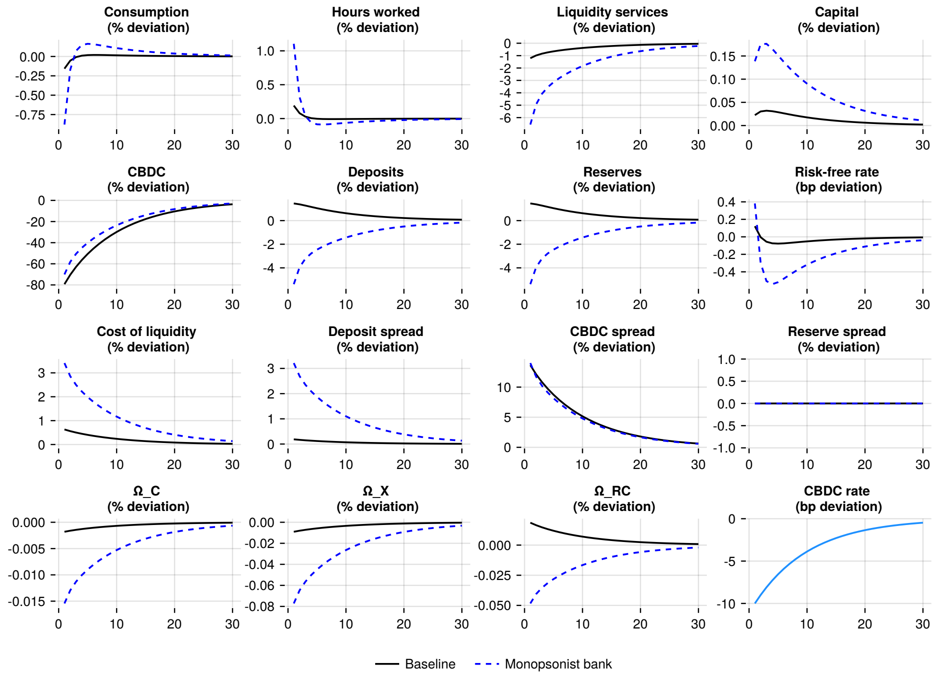


Figure 8: Higher steady state CBDC-deposit ratio $= \frac{1}{5}$

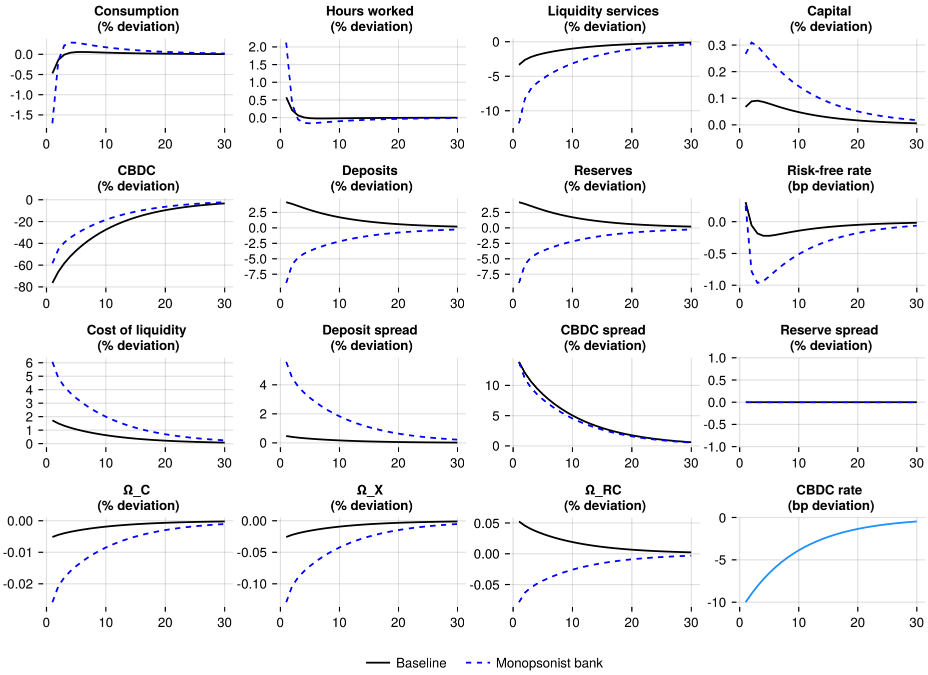


Figure 9: Lower CBDC-deposits substitutability $\epsilon = \frac{1}{5}$

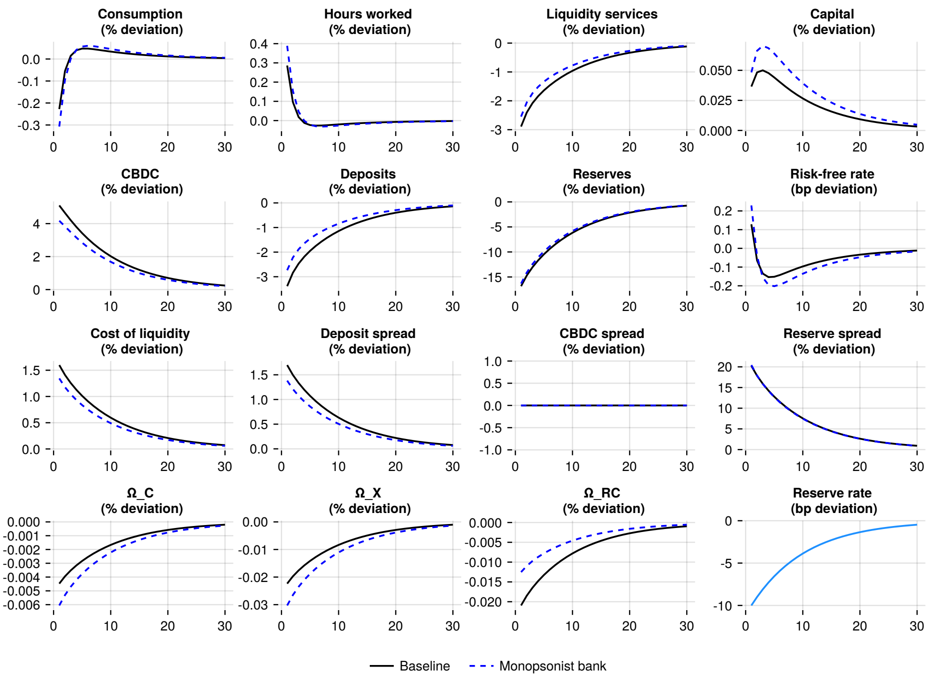


Figure 10: Higher CBDC-deposits substitutability $\epsilon = \frac{1}{7}$

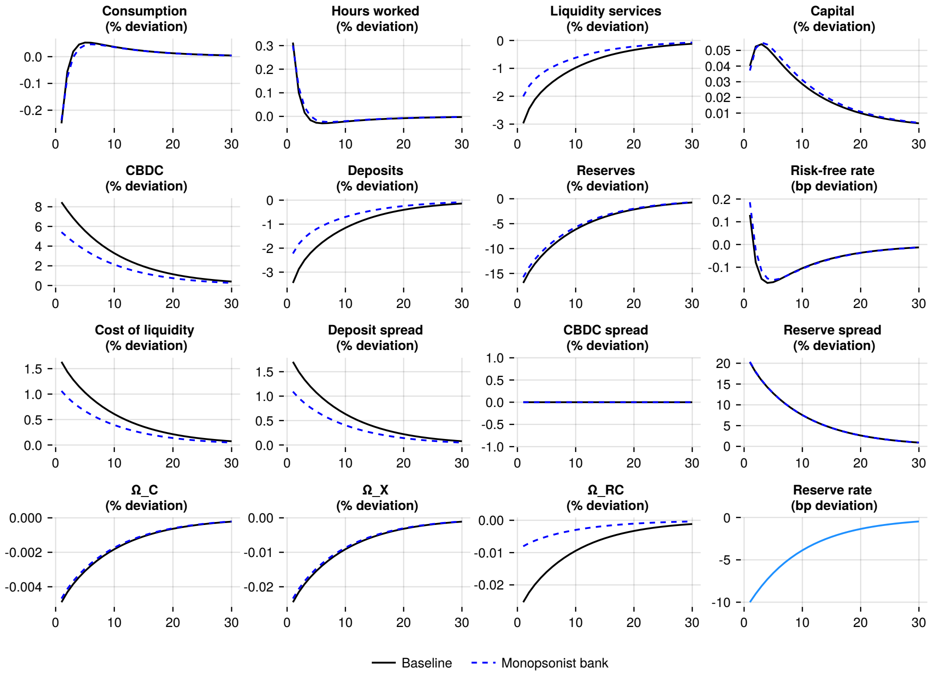


Figure 11: Lower steady state CBDC-deposit ratio $= \frac{1}{15}$

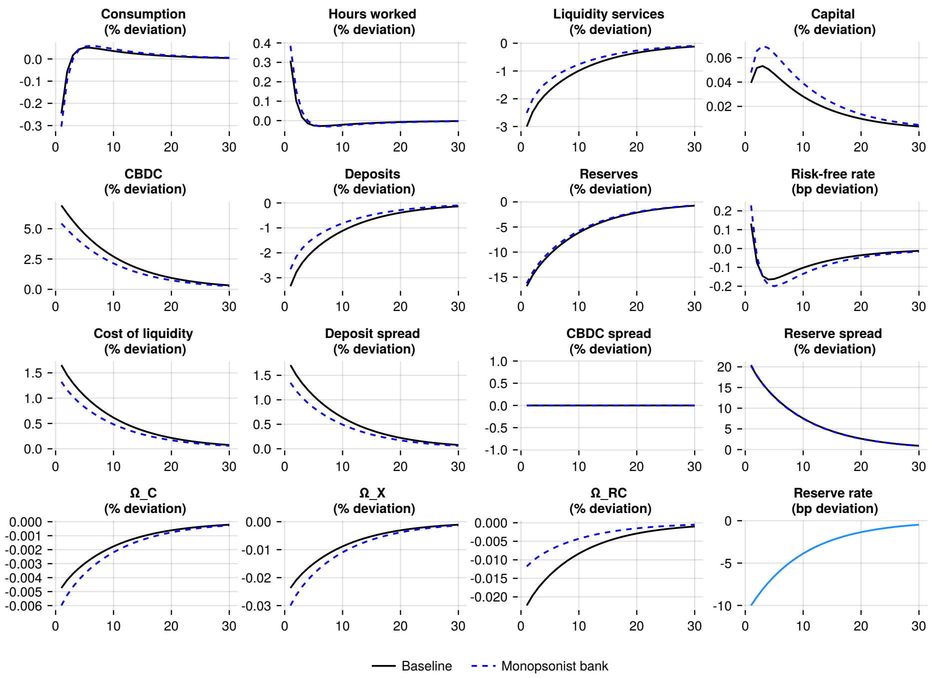
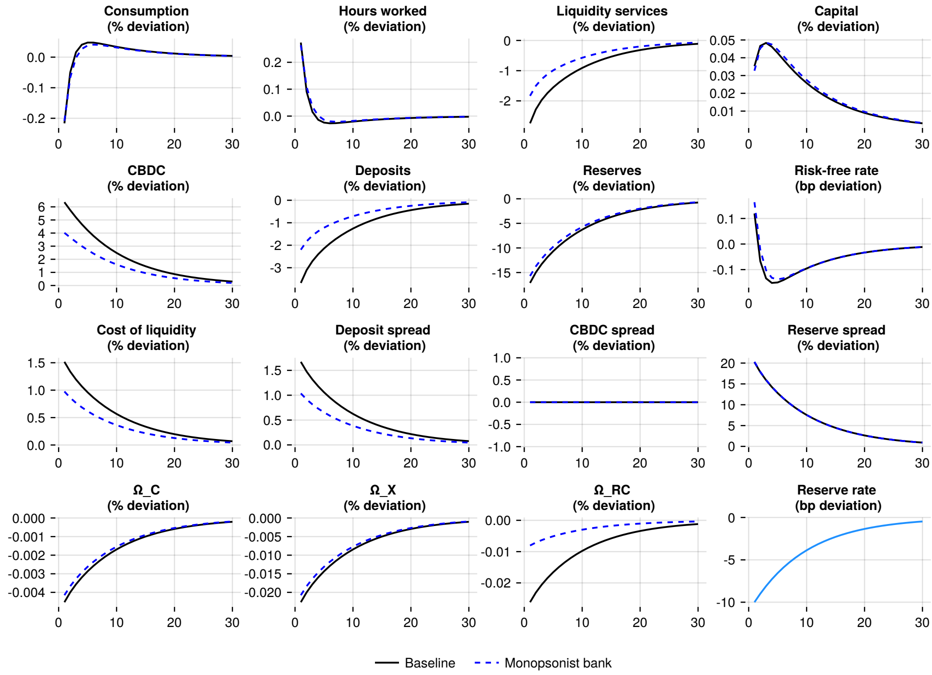


Figure 12: Higher steady state CBDC-deposit ratio $= \frac{1}{5}$



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